

A Re-examination of the Nineteenth Century French Influence on American Mathematics Education through Textbook Author Charles Davies

Sian Zelbo, Teachers College, Columbia University, New York

Abstract

Charles Davies was a prolific and influential nineteenth-century author of mathematics textbooks. Davies's 1835 adaptation of Bourdon's Éléments d'Algèbre is of particular interest because it forms part of a larger story about French influence on American mathematics education by way of the United States Military Academy at West Point. Prior studies have argued that French-American influence was undercut by the tendency of authors to alter the French analytic style of pedagogy for American audiences. This study adds to that line of research by giving more evidence of how this particular book originated and how the content was altered. It shows that Davies took credit for the translation of his subordinate, Edward Ross, and then simplified the book by removing modern mathematical concepts. Rather than spreading novel French ideas about mathematics, Davies's 1835 algebra book and its many subsequent editions likely reinforced the status quo of American mathematics education for decades.

Keywords: history of mathematics textbooks, history of mathematics education, Charles Davies, West Point, Louis Bourdon

Introduction

The French influence on American mathematics education in the nineteenth century by way of the United States Military Academy at West Point is well documented (see e.g., Molloy, 1975; Crackel, 2002). When Sylvanus Thayer took over as superintendent of West Point in 1817, he initiated a number of reforms that oriented the school towards the model of the *École Polytechnique*, including instituting a testing and ranking regime and requiring hours of French language classes each day so that the cadets could read French mathematics textbooks. Under Thayer's superintendency, Charles Davies served as the Professor of Mathematics. Because of his prominent position at West Point and his long career, Davies's influence on American mathematics education was far-reaching. Davies taught at least 780 cadets, many of whom taught mathematics themselves at West Point and other colleges around the United States after graduation. Davies also spread his influence by publishing his own textbook series, including adaptations of French works. The books sold millions of copies and dominated the American market in the nineteenth century.

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Although Davies's widespread influence on nineteenth-century American mathematics education is undeniable and may even be underestimated, we should exercise caution before concluding that the impact of Davies's books was to spread French influence and modernize American mathematics education. One of this era's most popular French-to-English adaptations, Davies's 1835 adaptation of Bourdon's 1817 *Éléments d'Algèbre*, was a poor vehicle for French influence. First, Bourdon published his original algebra textbook during a time of rapid change in mathematics and mathematics education in France, and the book largely reflected the thinking of an earlier era, with a lack of modern rigor, outdated notions of fundamental ideas, and a casual tolerance of contradictions. Even so, Bourdon's *Algèbre* was more modern in content than the English and American textbooks commonly in use in the United States at the time, such as the Charles Hutton textbook West Point had been using. Bourdon's *Algèbre* contained at least a glimpse of modern ideas of number and of functions. For all its deficiencies as a vehicle of modern mathematical ideas, a translation and adaptation of Bourdon's *Algèbre* therefore had the potential at least to expose vast American audiences to new European ideas.

Evidence presented in this study suggests, however, that Davies did not have expertise or interest in exposing American audiences to novel French ideas. Instead, Davies took advantage of the country's interest in French mathematics texts by publishing a translation that had just been produced by Edward Ross, an assistant professor who worked under him at West Point, under his own name. Davies altered and simplified Ross's text, rejecting much of what the French educational system had to offer the United States at that time. Davies's significant changes to Bourdon's work had the overall effect of making the book more concrete, lowering the substantive level of the material, and removing references to modern ideas. Rather than modernizing American mathematics education, the dominance of this single book in nineteenth-century American schools and universities may well have had the opposite effect.

Literature

There has been a persistent tendency, dating back to contemporaries of Davies, to mythologize West Point and overstate the impact of Davies's textbooks in providing "French methods" to generations of Americans. On the other hand, there has also been an undercurrent of commentary and scholarship recognizing Davies's mixed intellectual legacy, without disputing his vast influence. General George W. Cullum, an 1833 graduate of West Point and student of Charles Davies, compiled a register and biography of every graduate of West Point from 1802 to 1890, a document that also serves as a first draft of the school's history. In general, Cullum's attitude towards the school and its graduates is unsurprisingly reverent. Davies's particular entry was written by Jared Mansfield, another West Point mathematics professor and Davies's father-in-law. Mansfield is hyperbolic in his praise of Davies at times, especially in describing Davies's influence on generations of American mathematics students. Even Mansfield, however, admits that Davies's

textbooks "were not works of brilliant genius" (Cullum, 1868/1891, v. 1, p. 152) but says they were admirable in how they made the French mathematical texts accessible to ordinary people. Mansfield's comments are consistent with the assessment of Florian Cajori in his 1890 report to the United States government on the state of education at this time; Cajori acknowledges that improvements to American mathematics education were due to French influences (1890, p. 99), but about Davies in particular, Cajori says the reasoning in his books is "open to objection" (p. 121).

Subsequent scholarship has been mixed in its overall assessment about Davies and about West Point's French-influenced reforms. Lao Simons (1931, p. 117), David Eugene Smith (1834, p. 77), and Judith Grabiner (1977, p. 16) have all cited Davies as an important conduit of French influence on the United States without providing any caveats to that broad assessment. Karen Parshall and David Rowe, by contrast, have pointed out that the content of Bourdon's *Algèbre* was too elementary and dated at the time it was used in the United States to help the country catch up to the research communities that existed in France (1991, pp. 13-14). Others have argued that the potential for French influence was mitigated by Davies's desire to temper the French analytic style to make the book more "practical" for American audiences. Thomas Preveraud, for one, explains, "The analytical, radical and original French approach was progressively given up or softened by a so-called rigor of definitions more appropriate to methods of American and British algebras previously in use" (Preveraud, 2015, p. 7; see also Preveraud, 2016, pp. 24-27 and Pycior, 1989, pp. 137-142).

Gert Schubring and Amy Ackerberg-Hastings also note the intellectual deficiencies and incoherence of Davies's adaptations and place them in the broader context of Davies's business aspiration of creating a textbook series in his own name. Schubring points out that Davies's decision to publish his own adaptation of Legendre's *Éléments de Géométrie* (1794) when John Farrar's already existed, and to justify the need for the book with unconvincing criticisms of Farrar, makes sense when one understands that Davies needed a geometry book to complete his textbook series (Schubring, 2009, pp. 379-380). Amy Ackerberg-Hastings, with plenty of biographical evidence, portrays Davies as pompous, simple-minded, and lacking in academic integrity (Ackerberg-Hastings, 2020). She argues that Davies's impressive textbook sales and his influence generally were due to Davies's tendency to plagiarize other people's texts, his penchant for exaggerated self-promotion, and his publisher's aggressive marketing techniques. Ackerberg-Hastings also analyzes Davies teaching philosophy and concludes that it is characterized by the "intellectual incoherence" that results from borrowing other people's ideas (2020, p. 122) but also a simultaneous tendency to fall back on his own "concrete views" of mathematics when he could not reconcile those ideas (2020, p. 115).

This study adds to the body of research about Davies's influential adaptation of Bourdon's *Algèbre* by analyzing the origins of the book and the specific content changes Davies made to Bourdon's original work for American audiences.

Background About Davies and the Origins of his 1835 Book

The French reforms at West Point were part of a larger nineteenth-century movement in which American college professors, who in the eighteenth century had been heavily influenced by British, synthetic textbooks, were newly drawn toward the French analytic approach (Kidwell et al., 2008, p. 4). This interest in French mathematics education also coincided with the rise of the first "full-fledged American publishing houses" (Kidwell et al., 2008, p. 8-10). Davies therefore began his publishing career in a market highly favorable to American textbook series, and in particular those that included translations and adaptations of French works. Davies's adaptation of Bourdon was his fifth book in a series that already included a descriptive geometry text adapted from the works of Monge and Crozet and a geometry text adapted from Legendre. Including a French-based algebra text in his series was a logical next step for Davies in advancing his publishing career. Bourdon's book was an obvious choice as a source-book because just a few years earlier, Edward Ross had just published the most complete translation to date for use at West Point, and so Davies could use Ross's work as a starting point.

To understand the content choices Davies made in adapting Bourdon's *Éléments d'Algèbre*, it is first necessary to discuss the content of Bourdon's book. Louis Pierre Marie Bourdon (1779-1854) graduated from the *École Polytechnique* in 1798 as one of its earliest graduates. Bourdon's original 1817 textbook was a massive, 605-page text that began with the most elementary algebra material and extended into advanced topics such as exponential functions and infinite series. The book was well respected when it was written. Bourdon served as an admissions examiner for twenty years at the *École Polytechnique*, and parts of Bourdon's book were explicitly aligned with the admissions exam for the school. Bourdon's text was revised and expanded over several editions; the eleventh edition was published in 1856 after his death (Bourdon, 1856). But while *Algèbre* was thoroughly modern at the time it was written, French mathematics and mathematics education underwent rapid changes in the nineteenth century. Modern rigor and conceptions of intertwined, fundamental concepts such as number, continuity, and functions were percolating and working their way into French textbooks,¹ and Bourdon's early editions of *Algèbre*, on which Davies's text was ultimately based, reflected the inconsistencies of this era of transition. As this study will show through examples below, Bourdon was clearly aware of the changes around him, yet earlier ideas about fundamental concepts permeated his thinking.

¹ See Gert Schubring's *Conflicts between Generalization, Rigor, and Intuition: Number Concepts Underlying the Development of Analysis in 17-19th Century France and Germany* (2005) for a history of the fundamental concepts of number, variable, function, and limit in this time period.

Edward Ross was a graduate and assistant professor of mathematics at West Point when he produced his translation of Bourdon's *Algèbre* in 1831 for use at the school. Ross's translation seems to be based on the fifth edition of Bourdon's *Algèbre*, published in 1828, the most recent available edition when Ross did the translation. Ross's years at West Point as a student and then assistant professor of mathematics from 1817 to 1833 (Cullum, 1868/1891, v. 1, pp. 268-269) coincided precisely with Thayer's superintendency, and Thayer's admiration for French texts must have influenced Ross. Ross's book is a direct, sentence-by-sentence translation² of Bourdon, with the same mathematical examples and problems as the original. Ross does leave out three chapters of advanced material in his translation, along with pieces of two other chapters; he specifies in his preface which material was excluded (Ross, 1831, p. iii). Ross even retains Bourdon's section numberings, skipping numbers because of the omitted sections, to be as true as possible to the original. The only addition Ross makes is to insert a problem set, which he took (with credit) from the English author, John Bonnycastle. Ross's translation of Bourdon's *Algèbre* replaced an American adaptation of a more elementary textbook by British author Charles Hutton.

Charles Davies, like Ross, was a West Point graduate, but his tenure there as a student from 1813 to 1815 preceded Thayer's superintendency and Thayer's reforms, including the French language classes and French-influenced mathematics classes. In fact, Davies attended West Point when the scientific curriculum was "very limited" and well below the programs that were beginning to emerge at the country's best colleges (Crackel, 2002, p. 48). Nor was Davies self-educated at the time he entered West Point; he was only fifteen with "little education or acquaintance with the outside world" (Cullum, 1868/1891, v. 1, p. 152). Davies graduated after only two years, at the age of 17, before he had even completed his studies because he was needed in the army (Ackerberg-Hastings, 2000, pp. 222). Davies therefore had very little education when he was appointed as an assistant professor at West Point in 1817 at the age of 19. Davies was considered a skilled teacher, however, and was promoted to Professor of Mathematics in 1823; he remained at the school until 1837 (Ackerberg-Hastings, 2000, pp. 215-231).

At the time Ross published his translation, Davies was Ross's superior. Davies was West Point's only Professor of Mathematics at the time, overseeing Ross and the other assistant professors who did much of the teaching (Ackerberg-Hastings, 2000, pp. 227). Only two years after translating this book, Ross (who was a mathematics professor but also, technically, an army officer) was transferred out of West Point, and he was sent to fight in the country's wars in the West.³ In 1835,

² To clarify, this study does not attempt to judge the quality of the translation, but notes that it was Ross's intention simply to convey the meaning that was there and not to add or subtract from the ideas contained in the translation.

³ Ross spent six years in the army engaged in various operations and battles related to the United States' various wars against native Americans, including the forced displacement of Cherokee Indians to reservations in the West. After leaving the army, Ross taught mathematics at Kenyon College in

Davies used Ross's translation to produce his own adaptation and replaced it as the official algebra textbook at West Point. Davies did not credit Ross explicitly in his original edition; he only acknowledged in his preface that other translations existed, including the partial translations by Augustus de Morgan of University College London and Edward Ross (Davies, 1835, p. iii). In some subsequent editions, Davies modified the preface to give explicit credit to Ross but then dropped the credit and the reference to Ross altogether in later editions (e.g., Davies, 1835/1860).

Davies explains his motivation for adapting Bourdon's work in the beginning of the book; he says that the original is far too voluminous to cover in American schools and so the original needed to be abridged (1835, p. iii). His intention, he says, was to combine the best of "the scientific discussions of the French" and "the practical methods of the English" (1835, p. iv). In other words, he says, his aim was to pare down Bourdon's text and make it more practical. Davies held teaching jobs after leaving the army, including at the University of New York (today, New York University) and Columbia College (Ackerberg-Hastings, 2000, pp. 231-238; Cullum, 1868/1891, v. 1, pp. 151-152), but his real impact came from his textbooks. The Bourdon *Algèbre* adaptation was part of what became a series of books that stretched from elementary arithmetic through college-level course texts, which the publisher marketed as a "system" of learning that reflected "European methods of instruction" (e.g., "Good school books," 1850). A typical advertisement for Davies's books, shown in Figure 1 below, claims that because of its wide reach, the Davies textbook series "may be justly regarded as our national system of Mathematics" ("Good school books," 1850).

Similarly, the biographical register of West Point touts Davies's impact by claiming that Davies likely reached "millions of rising youths" through his textbooks and "made his mark on the educational system of this country probably quite as much, if not more, than any man in his generation" (Cullum, 1868/1891, v. 1, p. 152). These claims may not be a great exaggeration. One of Davies's publishers reported selling a total of 7,000,000 texts by Davies by 1875 (Rickey & Shell-Gellasch, 2010), and an 1870 comparative report on international engineering education lists Davies as one of the few authors whose textbooks formed the basis of the country's various engineering programs (*Education and Status*, 1870, p. 168). Clearly, to the extent that Davies's textbooks did reflect French advances in mathematics, the books could have been a powerful vehicle for transmitting those ideas to the United States.

Ohio and then taught mathematics and natural philosophy at the Free Academy in New York, the school that would become the City College of New York (Cullum, 1868/1891, v. 1, p. 269).

Good School Books for Good Schools.
PUBLISHED BY
A. S. BARNES & CO., NEW YORK,
AND
H. W. DERBY & CO., CINCINNATI.

DAVIES' SYSTEM OF MATHEMATICS.

THIS series, combining all that is most valuable in the various methods of European instruction, improved and matured by the suggestions of more than thirty years' experience, now forms the only complete consecutive course of Mathematics. Its methods, harmonizing as the works of one mind, carry the student onward by the same analogies and the same laws of association, and are calculated to impart a comprehensive knowledge of the science, combining clearness in the several branches, and unity and proportion in the whole. Being the system so long in use at West Point, through which so many men, eminent for their scientific attainments, have passed, and having been adopted as Text Books by most of the colleges in the United States, it may be justly regarded as our national system of Mathematics.

School and Academic Course.

DAVIES' PRIMARY TABLE BOOK. - Cloth back.
 DAVIES' FIRST LESSONS IN ARITHMETIC, Mor. back.
 DAVIES' SCHOOL ARITHMETIC, New edition, enlarged.
 DAVIES' ARITHMETIC, - Old edition, without answers.
 KEY TO DAVIES' SCHOOL ARITHMETIC, New edition.
 DAVIES' GRAMMAR OF ARITHMETIC.
 DAVIES' UNIVERSITY ARITHMETIC, 12mo, Sheep.
 Do Do Do Without answers.
 KEY TO DAVIES' UNIVERSITY ARITHMETIC.
 DAVIES' ELEMENTARY ALGEBRA, - Sheep.
 KEY TO DAVIES' ELEMENTARY ALGEBRA.
 DAVIES' ELEMENTARY GEOMETRY, - 12mo Sheep.
 DAVIES' PRACTICAL GEOMETRY & MENSURATION.

College Course.

DAVIES' BOURDON'S ALGEBRA, - 8vo Sheep.
 DAVIES' LEGENDRE'S GEOMETRY, - 8vo Sheep.
 DAVIES' ELEMENTS OF SURVEYING, - 8vo Sheep.
 DAVIES' ANALYTICAL GEOMETRY, - 8vo Sheep.
 DAVIES' DIFF. AND INTEGRAL CALCULUS, 8vo Sheep.
 DAVIES' DESCRIPTIVE GEOMETRY, 8vo Sheep.
 DAVIES' SHADES, SHADOWS, AND PERSPECTIVE, 8vo
 DAVIES' LOGIC OF MATHEMATICS, - 8vo.

Figure 1. 1850 advertisement for Davies's textbooks ("Good school books," 1850).

Content Choices Reflected in Davies's Adaptation

While the French origin of Davies's *Algèbre* adaptation was a selling point of the book, Davies adapted the book significantly to make it, in his view, more "practical" (Davies, 1835, p. iv). One effect was to make the text more elementary and more rooted in the ideas of the past. The examples below illustrate specific content choices Davies made in adapting Bourdon's work.

Davies replaces advanced content with more elementary content

The ordering and topics of the chapters of Davies's book coincide closely with those of Ross's book, as is evident from Table 1, below. This makes sense since Davies used Ross's translation as a starting point for his own. Ross had removed two full chapters of Bourdon in his translation (Chapter 4, *Analyse indéterminée* and Chapter 9, *Complément de la théorie des équations*), as well as certain articles from chapters 7, 8, and 10, and Davies removes this same material.

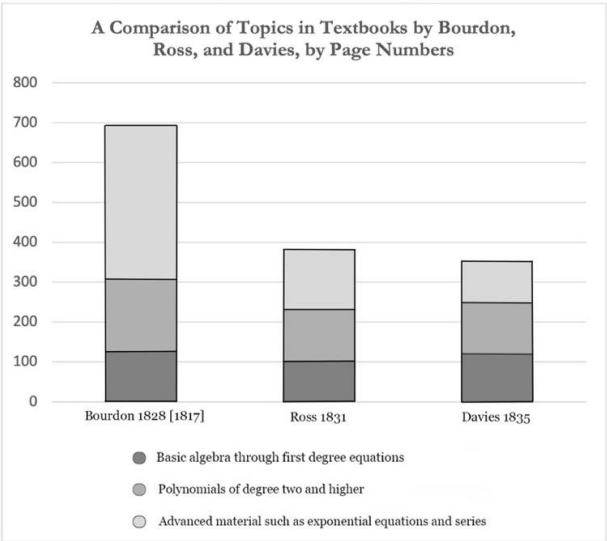
Table 1. Chapter contents for three algebra books

Bourdon 1828	Ross 1831	Davies 1835
Introduction	Introduction	Chap. 1
Chap. 1 "Des opérations algébriques"	Chap. 1 "Of algebraic operations"	"Preliminary definitions and remarks"
Chap. 2 "Des problèmes et des équations du premier degré"	Chap. 2 "Of problems and equations of the first degree"	Chap. 2 "Equations of the first degree"
Chap. 3 "Résolution des problèmes et équations du second degré"	Chap. 3 "Resolution of problems and equations of the second degree"	Chap. 3 "Extraction of the square root of numbers. Formation of the square and extraction of the square root of algebraic quantities. Calculus of radicals of the second degree. Equations of the second degree."
Chap. 4 "Analyse indéterminée"	Omitted	Omitted
	Inserted "Examples for Practice from Bonnycastle's Algebra"	Problems from Bonnycastle are inserted within the first 3 chapters, along with other material from the book (without attribution)
Chap. 5 "Formation des puissances et extraction des racines d'un degré quelconque"	Chap. 4 "Formation of powers and extraction of roots of any degree whatsoever"	Chap. 4 "Formation of powers and extraction of roots of any degree whatsoever"
Chap. 6 "Théories des progressions et des logarithmes"	Chap. 5 "Theory of progressions and logarithms"	Chap. 5 "Of progressions, continued fractions, and logarithms"
Chap. 7 "Théorie générale des équations"	Chap. 6 "General theory of equations"	Chap. 6 "General theory of equations"
Chap. 8 "Résolution des équations numériques à une ou plusieurs inconnues"	Chap. 7 "Resolution of numerical equations, involving one or more unknown quantities"	Chap. 7 "Resolution of equations"
Chap. 9 "Complément de la théorie des équations"	Omitted	Omitted
Chap. 10 "Complément de la théorie des suites"	Everything omitted except articles on figurate numbers.	Omitted

A superficial comparison of the chapters is misleading however because it does not reveal the extent of the changes Davies made to Ross’s translation. While Ross’s translation was already pared down from the original, Davies’s adaptation removes more advanced material. For example, Davies removed material on figurate numbers that Ross had included in Chapter 10. Additionally, Ross’s final chapters on the General Theory of Equations and Resolution of Numerical Equations, are reduced from 124 articles to 34.

In addition to these redactions, Davies makes smaller and more subtle changes to original to make the text more elementary. In Chapter 3, for example, Davies inserts several pages about the arithmetic of square roots (1835, pp. 120-133) before discussing roots of algebraic expressions. Davies also inserts more elementary background material about the arithmetic of continued fractions into Bourdon’s discussion of numerical approximations of logarithms (Davies 1835, pp. 265-273). In the early chapters Davies also replaces certain numerical and algebraic examples with simpler numbers and expressions. For example, when discussing first degree equations for the first time, Ross uses the example $2x + b = a$ (Ross 1831, p. 45); Davies changes the example to $x + x = 30$ (Davies 1835, p. 65), making the example both simpler and more concrete. Many of these changes are in the first few chapters of the book, suggesting that Davies relied on Bourdon’s judgment about how to present the more difficult material. The effect these changes over all is essentially to turn this American version of Bourdon’s *Algèbre* into a different, more elementary book than the original. Table 2, below, helps visualize the extent of the changes by showing how many pages each author dedicated to elementary versus advanced material.

Table 2. A comparison of topics in three textbooks by page numbers



Davies adapts the text to remove modern ideas about number

Bourdon's *Algèbre*, as mentioned above, was written during a time of change, and largely reflects the ideas of the eighteenth century. The textbook has no coordinate graphs, number lines, or other visual representations anywhere in the book that would suggest a continuity of real numbers from negative to positive infinity. Instead, Bourdon has a restrictive view of the idea of number as representing tangible quantities, which has implications throughout the book. Bourdon expresses his idea of number most clearly when he discusses negative numbers for the first time.⁴ He distinguishes arithmetic (which in his view did not allow for negative results) from algebra (which does) by saying that arithmetic "*portent toujours sur des êtres réels*" [always relates to real things] while in algebra "*on raisonne et l'on opère le plus souvent sur des êtres imaginaires*" [we reason and operate mostly on imaginary beings] (1817/1828, p. 93). Because of this view of negative numbers, Bourdon avoids them altogether in the beginning of the book and assumes without stating, in fact, that unknown quantities are positive.⁵ Negative numbers are introduced formally much later in the book, in the context of solving equations and interpreting a *result* that happens to be negative (Bourdon 1817/1828, pp. 86-87). Bourdon says that negative results often arising in algebra present "*des circonstances singulières qui embarrassent au premier abord*" [circumstances that are embarrassing at first sight] but can be explained (1817/1828, p. 86). For example, the equation $31 + x = 24$, gives rise to a supposed impossibility, an "*opération inexecutable*" [unworkable operation] of taking a larger number from a smaller one (1817/1828, p. 89). He explains the solution, $x = -7$, as meaning that the positive number 7 must be subtracted from 31 to get the result of 24. In other words, in this example Bourdon does not quite give -7 the status of a number in its own right. He reinterprets the solution in terms of an operation with positive numbers.

While Bourdon's conception of number reflects an earlier era, it is also clear that Bourdon was aware of a more modern view and introduces this view to his readers. Bourdon later gives an interpretation of negative numbers as numbers less than zero, explaining that this is the interpretation used by algebraists (1817/1828, p. 95). Bourdon demonstrates this idea by way of a pattern:

$$\begin{array}{cccccccccc} 6-1, & 6-2, & 6-3, & 6-4, & 6-5, & 6-6, & 6-7, & 6-8, & 6-9 \\ & 5, & 4, & 3, & 2, & 1, & 0, & -1, & -2, & -3 \end{array}$$

(1817/1828, p. 95; Ross 1831, p. 78). He concludes that -1 can be regarded as a number less than 0 and that -1 is greater than -2.⁶

⁴ See also Pycior (1989), pp. 139-142, which discusses Bourdon's and Davies's pedagogical treatments of negative numbers using the analytic/synthetic framework.

⁵ See p. 5 where he states that $x + b$ must be larger than x . Even in his treatment of series near the end of the book, Bourdon assumes without stating so that the common ratio must be positive (Bourdon, 1817/1828, pp. 315-326).

⁶ For a discussion of Bourdon's conflicted ideas about negative numbers and how his work fits into the broader history of the development of the concept of number, see Schubring, 2005, pp. 568-569.

Davies's readers, on the other hand, were not introduced to this modern view of negative numbers because Davies removed this second discussion from the book completely, leaving only the idea that negative numbers held a different, lesser status than positive numbers. Consistent with this view, Davies also alters Bourdon's definition of "subtraction" in the introductory section of the book in a way that leaves little room for understanding negative numbers as numbers less than zero. Bourdon gives two definitions for subtraction: (a) subtraction as diminishment and (b) subtraction as the difference of two numbers (Bourdon 1817/1828, pp. 1-2; Ross 1831 p. 1). The second definition is helpful for understanding the modern view of negative numbers because a number is not *diminished* by subtracting a number less than zero; yet, Davies removes this second definition (Davies 1835, p. 9). Davies reiterates his concrete view of number by writing later (in his own words): "If it were required to subtract 20 from 16, the subtraction could not be made by the rules of arithmetic, since 16 does not contain 20; nor indeed can it be entirely performed by Algebra" (Davies, 1835, pp. 104-105).

Davies adapts the text to remove the notion of a "function"

Given Bourdon's concrete idea of number and tendency to fall back on numbers that can be visualized, it should not be surprising that his algebra overall is static, with the letters mostly representing unknown constants rather than continuously varying numbers. By the time Bourdon wrote his algebra book, of course, the idea of the function and of the letters of algebra representing dynamic variables was hardly new (Schubring, 2005, pp. 19-22). At the *École Polytechnique* at that time, Augustin-Louis Cauchy was the professor of analysis and was giving the lectures upon which his 1821 *Cours d'Analyse* would be based. Still, ideas about functions took many decades to develop fully and then to seep down into school mathematics as a more elementary, fundamental concept. Pamphlets published regularly by the *École Polytechnique* detail the content of the school's annual admissions exams and reflect this period of transition. The 1828 admissions pamphlet makes no mention of functions (*Programme des Connaissances*, 1828). But by the 1850 admissions cycle, the list of topics had expanded to include calculus, and the concept of a "function" was included in that specific context. By contrast, the topic lists in trigonometry and analytic geometry material that year made no use of the term "function" (*Programmes pour l'Admission*, 1850, pp. 21-22).

The process of incorporating the idea of functions into the secondary school curriculum took even longer in the United States. American Herbert Hamley in his 1934 address to the National Council of Teachers of Mathematics, nearly a century after Davies published his 1835 algebra text, urged American educators to embrace the idea of the function to catch up to the pedagogical advances in Europe. He argued that while it was not clear when the idea of functions began to be taught in French schools, it was clear that the modifications of the program of the *École Polytechnique* were "always in the direction of increased emphasis on functionality" (Hamley 1934, p. 61).

Bourdon's original work was published in 1817, very early in this process of reform. Bourdon's *Algèbre* does include the concept of a function and in fact Bourdon uses the term over 200 times in his 1828 edition. However, the concept is treated as ancillary, not fundamental, as it was in the early admissions materials of the *École Polytechnique*. Bourdon introduces the definition of a function in the middle of the chapter on quadratic equations and invokes the idea to discuss techniques for finding maxima and minima. When Bourdon introduces the definition of a function, he hints that the idea is a new one in the algebra curriculum and that it is borrowed from higher levels of mathematics. He explains about the variable x , "*les analystes désignent quelquefois celle-ci sous le nom de variable indépendante . . .*" [analysts sometimes refer to it as the independent variable.] (1817/1828, p. 179).

Perhaps the starkest example of Davies's discomfort with modern ideas is his decision to remove many references to functions from Ross's book. As shown in Table 3 below, Ross uses the term "function" 87 times in his translation of Bourdon, only a fraction of Bourdon's 227. This difference reflects the fact that Ross eliminated some of the more advanced material from Bourdon's book, as discussed earlier. Davies, on the other hand, uses the term only 22 times in his 320-page textbook, which represents something different – a deliberate choice to remove references to the concept even within the chapters he kept. Davies leaves out Bourdon's section on maxima and minima entirely, where Bourdon defines "function." Davies instead introduces the term "function" formally later when discussing geometric series. He states that before explaining the techniques for working with geometric series, "it will be necessary to explain what is meant by the term *function*" (1835, p. 241). Davies defines the term "function," however, in a way that is not particularly useful because it does not include the idea of an independent and dependent variable. Instead, Davies explains that in the equation $a = b + c$, each of a , b , and c "mutually depend on each other for their values" and so each is a function of the other (1835, p. 241). The letters Davies is referring to here are constant coefficients of a geometric series, not variables. After giving this curious definition of "function," Davies never invokes the concept of a "function" again in his discussion of geometric series and then proceeds to use the word only sporadically throughout the book. For example, Davies adapts the first part of the chapter on the general theory of equations to his purposes by replacing every use of the word "function" with "polynomial." He does so inconsistently, however, and uses the word "function" for the first time much later in the chapter in a context that seems almost random. These vestigial uses of the term "function," dwindled down to 10 in the 1877 edition of Davies's book, published just after his death (Davies, 1835/1877).

Table 3. Uses of the word "function" in various editions of three textbooks

Book	Number of times the word "function" or "functions" was used
1828 Bourdon	227
1831 Ross	87
1835 Davies	22
1837 Davies	30
1838 Davies	19
1842 Davies	19
1845 Davies	25
1847 Davies	25
1848 Davies	24
1850 Davies	23
1851 Davies	25
1852 Davies	25
1853 Davies	25
1854 Davies	15
1857 Davies	14
1859 Davies	14
1860 Davies	14
1867 Davies	14
1868 Davies	14
1871 Davies	13
1875 Davies	10
1877 Davies	10

Conclusion

Translations of French textbooks, including Davies's adaptation of Bourdon's *Algèbre*, circulated widely in nineteenth century America. As a vehicle for French ideas, the books had the potential to elevate the level of mathematics content in the United States. Davies took advantage of the country's interest in French mathematics by including of an adaptation of Bourdon's *Algèbre* in his textbook series. And yet, ironically, Davies's changes to Bourdon's *Algèbre* undercut the French ideas he purported to spread. Given Davies's modest background in mathematics, it is possible that he did not fully engage with the more advanced material in the Ross text he was working from. Davies's haphazard removal of references to functions and his deference to Ross in the later chapters of the book suggest that this may be true. Either way, the changes Davies made resulted in a book that was more elementary, more dated, and more intellectually incoherent. West Point, because of its prestige, had an influence on institutions all over the United States, and that prestige helped sell Davies's books. Rather than advancing American mathematics, the popularity of Davies may well have set it back decades.

References

- Ackerberg-Hastings, Amy (2000). Mathematics is a gentleman's art: Analysis and synthesis in American college geometry teaching, 1790-1840 (Doctoral dissertation, Iowa State University).
- Ackerberg-Hastings, Amy (2020). Charles Davies as a philosopher of mathematics education. In M. Zack & D. Schlimm (Eds.), *Research in History and Philosophy of Mathematics. Proceedings of the Canadian Society for History and Philosophy of Mathematics/ Société Canadienne d'Histoire et de Philosophie des Mathématiques* (pp. 1-15). Cham, Switzerland: Birkhäuser.
- Bourdon, Louis (1817). *Éléments d'Algèbre*. Paris: V. Courcier.
- Bourdon, Louis (1828). *Éléments d'Algèbre* (5th edition). Paris: Bachelier. (Original work published in 1817.)
- Cajori, Florian (1890). *The Teaching and History of Mathematics in the United States*. Washington, D.C.: Government Printing Office.
- Crackel, Theodore J. (2002). *West Point: A Bicentennial History*. Lawrence, Kansas: University of Kansas Press.
- Cullum, George W. (1891). *Biographical Register of the Officers and Graduates of the United States Military Academy at West Point, N.Y* (3rd edition). Boston & New York: Houghton, Mifflin, & Co. (Original work published in 1868.)
- Davies, Charles (1835). *Elements of Algebra, Translated from the French of M. Bourdon*. New York: Wiley.
- Davies, Charles (1860). *Elements of Algebra on the Basis of M. Bourdon*. New York: A. S. Barnes & Burr. (Original work published in 1835.)
- Davies, Charles (1877). *Elements of Algebra on the Basis of M. Bourdon*. New York & Chicago: A. S. Barnes. (Original work published in 1835.)
- Education and Status of Civil Engineers, in the United Kingdom and in Foreign Countries*. (1870). London: Council of the Institution of Civil Engineers.
- Good school books for good schools. (1850, June 20). *Indiana State Sentinel* (Indianapolis), p. 3.
- Grabiner, Judith V. (1977). Mathematics in America: The first hundred years. In D. Tarwater (Ed.), *The Bicentennial Tribute to American Mathematics, 1776-1976* (pp. 9-24). Washington, D.C.: Mathematical Association of America.
- Hamley, Herbert R. (1934). *Relational and Functional Thinking in Mathematics*. New York: Teachers College, Columbia University.
- Kidwell, Peggy A., Ackerberg-Hastings, Amy, & Roberts, David L. (2008). *Tools of American Mathematics Teaching, 1800-2000*. Baltimore: Johns Hopkins University Press.
- Molloy, Peter M. (1975). *Technical Education and the Young Republic: West Point as America's École Polytechnique, 1802-1833* (Doctoral dissertation, Brown University).
- Parshall, Karen & Rowe, David (1991). *The Emergence of the American Mathematical Research Community 1876-1900: J. J. Sylvester, Felix Klein, and E. H. Moore*. Providence, RI: American Mathematical Society.
- Preveraud, Thomas (2015). American mathematical journals and the transmission of French textbooks to the USA. In K. Bjarnadóttir, F. Furinghetti, J. Prytz & G. Schubring (Eds.), *"Dig where you stand" 3. Proceedings of the third International Conference on*

- the History of Mathematics Education* (pp. 293-308). Uppsala: Department of Education, Uppsala University.
- Preveraud, Thomas (2016). Les Etats-Unis, espace de compromis. L'adaptation des algèbres françaises de Lacroix et Bourdon aux usages domestiques (1818-1835). *Revue d'Histoire Des Mathématiques*, 22(2), 185–221.
- Programme des Connaissances Exigées pour l'Admission a l'École Polytechnique*, 1828. (1828). In A. Fourcy, *Histoire de l'École Polytechnique* (p. 375). Paris: l'École Polytechnique.
- Programmes pour l'Admission et pour l'Enseignement a l'École Polytechnique*. (1850). Paris: Imprimerie Nationale.
- Pycior, Helena M. (1989) British synthetic vs. French analytic styles of algebra in the early American republic. In D. E. Rowe & J. McCleary (Eds.), *The History of Modern Mathematics, Volume 1: Ideas and Their Reception* (pp. 125-156). San Diego, CA: Academic Press.
- Rickey, V. Frederick & Shell-Gellasch, Amy (2010). Mathematics education at West Point: The first hundred years – Charles Davies, mathematics professor, 1823-1837. *Convergence*. Retrieved from <https://www.maa.org/press/periodicals/convergence/mathematics-education-at-west-point-the-first-hundred-years-charles-davies-mathematics-professor>
- Ross, Edward C. (1831). *Elements of Algebra, Translated from the French of M. Bourdon*. New York: Clayton & Van Norden.
- Schubring, Gert (2005). *Conflicts between Generalization, Rigor, and Intuition: Number Concepts Underlying the Development of Analysis in 17-19th Century France and Germany*. New York: Springer.
- Schubring, Gert (2009). A origem da geometria de Legendre e o seu impacto internacional. In L. C. Guimarães (Ed.), *Adrien-Marie Legendre, Elementos de Geometria. Reedição da Primeira Tradução Brasileira de 1809* (pp. 353-384). Rio de Janeiro: Editora LIMC.
- Simons, Lao G. (1931). The influence of French mathematicians at the end of the eighteenth century upon the teaching of mathematics in American colleges. *Isis*, 15(1): 104–123.
- Smith, David Eugene & Ginsberg, Jekuthiel (1934). *A History of Mathematics in America before 1900*. Chicago, IL: Mathematical Association of America / Open Court Publishing.