Concepts of mathematical giftedness in German schools: A cultural-historical approach

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Abstract

The concept of mathematical giftedness and, accordingly, its promotion has been dealt with in different ways in Germany over the last 150 years. Institutional support has been shaped by economic conditions as well as by socio-political and cultural-historical factors, and by traditions of different thought collectives. We show that characteristics of the attitudes of the interest groups in mathematics education[1] in the sense of Paul Ernest can also be explained by cultural-historical factors.[2]

Keywords: cultural history, promotion of mathematical giftedness, Germany, interest groups

Different concepts of mathematical giftedness

Over the past 200 years, different conceptions of school mathematics in the *Gymnasium*, such as elementarizations of higher mathematics, tool for science, main school subject and selection subject, character school, popular sport, or competitive sport have led to different roles of mathematics teachers in the discovery and promotion of mathematical talent. Different conceptions of mathematical giftedness and of the conditions under which successful mathematical activities become possible are linked to different social practices. The latter partly dependent on educational policy frameworks, but they are mainly shaped by their cultural and social history.

The study of mathematical giftedness has a long tradition and many facets. From a historical and philosophical perspective, mathematical giftedness can be related, for example, to famous mathematicians and their schools of thought. The promotion of scientific and technical elites plays a role in political economy contexts, whereby selection procedures such as IQ tests enable the utilization of "predisposed" individual mathematical abilities. In psychology, approaches in which structure and formal reasoning are particularly salient and visible serve to model thought processes. Sociology asks, among other things, whether the promotion of predispositions that we call mathematical giftedness is socially just and benefits not only the individual but also society. These and other aspects enter

[1] For further explanation see Paul Ernest's Philosophy of mathematics education (Ernest, 1991).
[2] Parts of this chapter can be found in the Chapter *Meine beste Schülerin, mein bester Schüler* (My best pupil) (Weiss & Kaenders, 2021) and in Begabungsförderung aus kulturhistorischer Sicht (A historic – cultural perspective on promotion of giftedness) (Weiss, 2022), which focuses on pedagogical issues and the role of the teacher in the promotion of giftedness.

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into our treatment of the concept of mathematical giftedness, sometimes reflectively, but mostly implicitly through established practice[3].

The way we talk about mathematical giftedness and we deal with it depends also on the pedagogical theoretical framework, in which the concept is developed.

In our society, the ability to do mathematics is strongly related to the concept of giftedness. From a cognitivist point of view, giftedness is usually attributed to the learner as a personal disposition and the promotion of giftedness is understood as the promotion of this disposition of the individual. The constructivist view of mathematical giftedness also sees it as the potential of the child; as a result of individual support, every child can become gifted in its own way. Both perspectives focus all attention on the child.[4] From the perspective of the cultural-historical school founded by Lev Vygotsky, giftedness occurs in a collaborative social practice of teacher and student (or more generally expert and novice)[5]. From this perspective, to be considered gifted is to be recognized and chosen by an expert. Promotion then consists in the development of a relationship that provides insights into values and norms of a social practice. This relationship is thus also shaped by the teacher's ideas about success and successful participation in this social practice and her role as a role model (Wenger, 1998). The concept which considers giftedness as a relationship between teacher and gifted student focuses on the existing social practice. Pedagogical theories related to this approach are the cultural-historical school (Vygotsky, Luriya, Leontyev) and the theory of situated learning - the community of practice (CoP)[6].

The focus of this paper is on the concept of giftedness in school use, more specifically in German secondary and high schools. In order to better understand the related school practice, we take a look at more than a century of traditions in which contemporary notions of mathematical giftedness, applied in everyday school life, have developed. The description of the dealing with mathematical giftedness in a school context over a longer period of time that is marked internationally and nationally by tensions, contradictions, and far-reaching social, cultural, economic, and technical changes cannot be portrayed through a single story. As indicated earlier, such a description also depends heavily on contextualization and pedagogical concepts.

In the following, we will focus on raising awareness of the use of the concept of mathematical giftedness in order to contribute to a better understanding of current school-based approaches to promotion of mathematical giftedness. We

[3] Analyses of the concept of giftedness from a social science perspective can be found, among others, in Böker, Horvath & Kenneth (2018)

[4] See e.g. (Leikin, et. al. 2017).

[5] See also Cole, Michael: The zone of proximal development: Where culture and cognition create each other, in: Wertsch, James V. (Ed.): *Culture, communication, and cognition: Vygotskian perspectives*. Cambridge 1985, pp. 146–161.

[6] The theory of CoP applies to any practice, during the last years it was mostly used in economical contexts (Lave & Wenger, 1991).

want to use the notion of mathematical giftedness with caution and precision, clarifying or sidestepping its troubling connotations and implications.

One perspective that takes into account cultural-historical aspects in the context of knowledge is Ludwik Fleck's theory of thought styles and -collectives.

In contrast to the model of the CoP, the perspective of the thought collective focuses on the transmission of and dealing with knowledge, in particular with scientific knowledge and its institutional dissemination. A thought collective is defined as "a community of persons mutually exchanging ideas or maintaining intellectual interaction", whereby "it also provides the special "carrier" for the historical development of any field of thought, as well as for the given stock of knowledge and level of culture." (Fleck, 1935, p. 29)

> The ... thought style is the result of theoretical and practical education of the given person, and passing from the teacher to the student, it represents a certain traditional value, subject to a specific historical development and specific sociological laws. (Fleck, 2011, p. 219)[7]

Even if the school lessons are not directly dedicated to teaching modern scientific mathematical knowledge, the promotion of gifted students can be seen as a binding link between the esoteric circle, consisting of scientists and experts of the field and the exoteric circle of interested laity of the thought collectives. (Fleck, 1935, p. 105).

The critical questioning of basic principles of mathematics education is a main concern of critical mathematics education. Our perspective allows to see current mainstream ideas about the concept of mathematical giftedness and the promotion of mathematical giftedness in a cultural-historical context and thus give a space to other concepts of giftedness.

> …, whether we like it or not, we can never sever our links with the past, complete with all its errors. It survives in accepted concepts, in the presentation of problems, in the syllabus of formal education, in everyday life, as well as in language and institutions. Concepts are not spontaneously created but are determined by their "ancestors". That which has occurred in the past is a greater cause of insecurity-rather, it only becomes a cause of insecurity – when our ties with it remain unconscious and unknown. (Fleck, 1935, p. 20)

The cultural-historical perspective allows to put implicitly existing ideas deviating from the mainstream into a tradition[8].

Mathematical giftedness and mathematical performance

State interest in the institutional promotion of mathematical giftedness grew in Germany as a result of industrialization in the 19th century. The establishment of polytechnic schools and technical colleges for the education of mathematically and scientifically broadly trained technical specialists and engineers was accompanied by the founding and strengthening of higher schools, which, in contrast to the old-

[7] Translation by the author.

[8] See also (Münch & Przybilski, 2017)

language grammar schools[9], placed an emphasis on mathematical and scientific education - the *Oberrealschule* and the *Realgymnasium* (Toepell, 2003).

In this process, the concepts of giftedness and achievement as fundamental terms of education and educational policy, in the school context came into opposition conditionally.

> While "achievement" is commonly associated with will, diligence, and perseverance, giftedness, in terms of linguistic history, refers to a good acquired without one's own efforts or exertion. (Kössler, 2018, p. 193)

In the sense of the humanistic educational ideal, to be mathematically gifted meant to have a potential for achievement - a strongly developed ability to grasp relationships and the essential in new more general situations (Knabe, 1960). In terms of mathematics, this meant possessing exceptional analytical and structural thinking skills for grasping logical relationships. Together with the widespread view of giftedness as predisposed or even inherited, the gifted mathematician (and here it was almost exclusively men[10]) was attributed self-discipline, perseverance and stamina, but this less as a prerequisite, but rather resulting from the occupation with mathematics.

The development of applied mathematics and knowledge and skills, which were considered success factors for successful technical and scientific studies, led to a broader, and in some cases contradictory, conception of successful mathematical activity and achievement, and thus also to new concepts of mathematical talent.

The image of the brilliant technician and engineer also included, for example, broad and transferable knowledge and skills in mathematics and the natural sciences, which the latter had to acquire primarily through diligence and training. In his description of the development of mathematics in the 19th century, Felix Klein (1926) goes into great detail about the diversity of talents, ways of working, and interests of well-known mathematicians, leading to typifications of mathematical ways of thinking[11].

The fact that also in the school context the notion of mathematical giftedness was shaped by the achievements of successful researching mathematicians, but also by their self-images and ideals, can be explained, among other things, by the progressive academization of the teaching profession *Gymnasiallehrer* in the 19th century. Thus, the teachers, professors, and principals working in the *Gymnasium* or other types of *Höheren Schulen* had studied mathematics, often had a doctorate in mathematics, and were often still engaged in research, mostly elementary mathematics. Teaching materials, such as models, task collections, and

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9 In the *altsprachlichem Gymansium* as an result of the Humboldtian Reform the focus was on Old greeke and latin and mathematics as a platonic subject.

10 Women's mathematical achievements, such as those of Sophie Germain and Sophia Kovalevskaya, are portrayed as degenerations that prevent female happiness. See e.g. (Möbius, 1902).

11 Stimulated also by modern developments, the emergence of new mathematical disciplines and the crisis in the foundations of mathematics, many mathematicians engaged in research are concerned with the history and philosophy of mathematics as well as the peculiarities of mathematical thinking. Cf. also (Poincaré, 1914).

mathematics textbooks, were thus developed and used by mathematicians for whom thinking about mathematical giftedness involved very personal and emotional issues. If students could quickly follow conceptual thoughts, transfer them to other representations, or creatively vary and generalize solutions, these were precisely the activities that were also essential to the successful inquiring mathematician. Both the 19th century reform of the teaching of geometry, *Neuere Geometrie*,[12] and the *Meraner Reform* of the Gymnasium mathematics teaching were aimed at promoting experimental approaches and working in different representations and contexts. Clarity, functional, process-related thinking, representation and use of the mobility and relatedness of objects, application-related contexts, inductive concept development oriented to paradigmatic examples and, above all, the self-activity of the students were leitmotifs of the *Meraner Reform*. The Meran conception of *Gymnasium* mathematics education made it possible to see student activity and mathematical achievement in a unified way; mathematical work was carried out at reduced problems accessible to the students, with the goal of a general training of functional thinking. The intended mathematical activity of pupils was similar to that of the researching mathematician, except that it dealt with less complex issues.

Aside from that, the role of the Matura as an entrance examination for technical colleges and universities implied, that the curricula of the *Gymnasium*, *Realgymnasium* and *Oberrealschule* were primarily characterized by preparation for university studies (Wolter, 2016). For mathematics classes, the latter meant the introduction of differential and integral calculus, as well as the strengthening of analytic geometry, where good performance could also be achieved by mastering calculus and mathematical terminology - that is, through diligence and repetition.

Since the end of the 19th century, the state's interest, and thus that of the school as an institution, in selection procedures based on performance criteria and opportunities for promoting future elites within the framework of secondary schools grew (see also Heinemann, 2021).

> As the privileges of birth became less important since the Enlightenment and societies were designed in a new meritocratic way, based on performance, the potentials, talents and abilities of individuals became more important, their talents and abilities gained social significance. Talent and intelligence established themselves as fundamental concepts in the measurement and classification of people and in the allocation of educational opportunities and professional careers. (Kössler, 2016, p. 103)

It was primarily the economic interests in the exploitation of mathematical talent that, already in the 19th century and increasingly in the 20th century, led to the recognition, assessment, and typification of mathematical talent becoming the focus of attention and linking the concept of achievement to performance in

[12] See (Kitz, 2013, pp. 139-149) or (Weiss, 2018) for implementations of the German reform of geometry teaching and the Meran reform (a reform of German mathematics education in the decades following the Meran Conference in 1905).

testing procedures (Stern & Wiegmann, 1922, Heinemann 1921). Intelligence research, which developed rapidly in the early 20th century, also focused on the development of tests and selection procedures rather than on understanding and dealing with the phenomenon of "mathematical giftedness"(Graumann, 2014, Lerch, 2021). For high school mathematics education, governmental interest was expressed in the development of assessment systems of school performance in the subject of mathematics to evaluate mathematical giftedness.

> In school language, a child is called mathematically gifted if his or her performance in that subject is good or very good without special effort.(Knabe, 1960, p. 252)

The "effortless" achievement can be seen here as an expression of the viewpoint of seeing mathematical giftedness as a given disposition independent of instruction. That teachers were aware of the oversimplification of this view is shown by the doubts expressed in the same section of the *Handbuch der Schulmathematik* (Handbook of School Mathematics) as to whether the performance observed in mathematics lessons can also demonstrate the ability to work scientifically and productively in mathematics:

> School instruction in mathematics is designed so that student average can show performance, ... The achievements in mathematics do not require, apart from a certain intelligence, which is necessary for the higher school in the first place, no special aptitude and further involvement with school material. (Knabe, 1960, p. 252)

The approach proposed in this situation encouraged teachers to fulfill their responsibility by gaining a better understanding of the existing strengths of the pupils in order to promote them accordingly on an individual basis:

> If the teacher wants to make a judgment about real aptitude, perhaps to check the ability to do scientific work, deeper observations about performance and the student's behavior are necessary. (Knabe, 1960, p. 253)

Examinations were, of course, used to assess performance. In his methodology for teaching mathematics in high schools and for textbook design, which was authoritative until the 1950s, Walter Lietzmann argued for performance examinations that were performed by teachers knowing the students well and for a long time:

> That such examinations, where examinee and examiner know each other, are the rule in our country, seems to me to be an essential advantage of our school system. Where, as in France or in England, both parts are strangers to each other or even - when limited to written examinations - do not meet at all, the examination itself is more difficult. Nevertheless, it does not lead to higher performance on average, despite a sometimes undignified exam drill. (Lietzmann, 1926, p. 207)

For the mathematics teacher, then, the oral exams were seen as better opportunities for performance assessment, since in the interaction mathematical giftedness became perceptible and could be experienced. As it was also expressed in the guiding ideas of the *Meraner Reform*, the distinctions between the student's alternately guided by an experienced mathematician and independent performed mathematical activities and the work of the researching mathematician lay more in the objects of investigation than in the activities themselves. Oral examinations were thus seen as conversation of a student with an expert, whom he knew from class and to whom he could demonstrate a conceptual understanding of what he had learned. Fostering mathematical talent thus also consisted of conceptualizing and teaching elementarized problems from higher mathematics.

In addition to challenging mathematical problems at school level, which gave the teacher the opportunity to observe students' procedures, psychological criteria were increasingly given to the teacher. These were often observable on the basis of different performances and were therefore easier to assess than mathematical problem-solving processes of individual students in a heterogeneous class[13]. The development and also implementation of the testing of such criteria was more and more taken over by specialized psychologists.

> A new approach was taken for the first time (1917) in the selection of students for the Berlin schools for the gifted. On the basis of extensive tests, each taking several days, with the means of experimental psychology, the abilities of the pupils were measured and then compared mathematically. The following were determined: Attention and concentration ability, memory, combination, conceptual range, judgment, visualization, and observational skills. (Lietzmann, 1926, p. 210)[14]

The formation of gifted classes also countered the problem of achievement heterogeneity. The growing responsibility of mathematics teachers at secondary schools for recognizing and promoting mathematical talent as a preparation for a successful career in mathematics, science, or technology was not limited to secondary schools with a mathematics and science focus. The important role of technology in Germany's hegemonic aspirations on the eve of and during World War II also led to a new importance of mathematics in the educational canon. Mathematics education should not only teach the abilities and skills that were the basis for careers in engineering, science, business, commerce, and administration; it also served as a school of character.

> But the far more important thing is the educational value that follows from the spiritual connection of mathematics with the Third Reich. The basic attitude of both is the heroic one. [...] Both demand service: mathematics the service to truth, sincerity, accuracy. [...] Both are anti-materialistic. [...] Both want order, discipline, both fight against chaos, arbitrariness. (Hamel, 1933, p. 307)

Girls and women, especially of the lower social classes, were also assigned a new role through mathematics in Nazi Germany. In mathematics books for girls, bookkeeping was presented not only as a means to effective housekeeping, but also as a service to the Führer, the people, and the Fatherland. Thus reads an

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[13] See also (Stern, 1922), (Heinemann, 2021), (Lerch, 2021).

[14] See (Schregel, 2021) about German schools for gifted at the turn of the century and (Moede et. al., 1919) for Berlin`s first school for gifted children.

introductory text to mathematics tasks in an arithmetic book for girls, quoting Hindenburg:

> After the great duties are fulfilled, the housewife can take care of the daily needs. Diligence and industriousness create the means for the prosperity of the nation's prosperity and its freedom, but only if they are managed sparingly. Without thrift even diligence and industriousness do not bring full success, Hindenburg. (Specht, 1936, p. 26)

The tendency of intelligence research to describe giftedness by psychological characteristics had already led to type theories of mathematical talent since the beginning of the 20th century.[15] In Nazi Germany, Ludwig Bieberbach, following such a type theory of the psychologist Erich Rudolph Jaensch, developed the theory of a German mathematics and an anti-Semitic typology of mathematical styles (Mehrtens, 1989, pp. 205-216).

After the end of the war, aptitude and intelligence gained a new significance in the context of elite recruitment in the new democracy, which had abolished the remaining privileges of birth. In his article "In Search of an End to Stupidity," Kössler (2018) describes the debates on giftedness since World War I through two tendencies that were connected in contradictory ways. On the one hand, giftedness selection was seen as the starting point of a necessary rationalization of state and society;

> giftedness had to be systematically recognized, selected and used according to the needs of society, so that Germany could survive in the Darwinian competition between nations. (Kössler, 1989, p.111)

Individualized tests for vocational selection, which aimed at a great adaptation of abilities to existing economic needs, were supposed to guarantee higher performance but also higher satisfaction of the performers. On the other hand, orientation toward the achievement principle also aroused fears that the excessive promotion of gifted children from lower classes could threaten the socio-moral order of society.

Promotion of mathematical giftedness in West Germany

In the first years of the postwar period of the Second World War, mathematics education in the old federal states was again strongly oriented toward the ideas of the Weimar Republic. The three-tier school system was retained despite the guidelines of the Allies. Alexander Israel Wittenberg (1926-1965) represented a new approach to a general mathematics education that aimed to improve the participation of all citizens in society and to educate them to become critically thinking, responsible citizens (Wittenberg, 1957). His pedagogy was oriented toward holistic individual development and operated accordingly with a concept of giftedness that did not reduce intelligence to the ability to think rationally and to selected personality traits. However, these general education aspects were soon superseded by the goals of the mathematics education reform New Mathematics,

[15] See e.g. (Thorndike, 1936) and (Stern & Wiegmann, 1922) as well as (Strunz, 1956).

which saw higher education primarily as a means of preparing students for careers, specifically for technical and mathematical-scientific fields of activity.

The long-term educational goals formulated at the 100th plenary session of the *Kultusministerkonferenz* (Standing Conference of the Ministers of Education and Cultural Affairs of the Länder) in 1964 included the education of every individual to the highest level of his or her ability, the creation of educational opportunities that were more closely geared to the aptitude of the individual, and measures to place students in educational programs that were appropriate to them (e.g., observation levels).

> The development of "talent reserves," the improvement of the "selection process in the transition from elementary school to junior high and high schools and the consolidation of the network of these schools was the focus of a broad-based strategy of educational planning and policy. (Führ & Furck, 1998, p.15)

The educational reform, known internationally as New Math, was initiated and organizationally supported primarily by the economic organization OECD (at that time still OEEC).

For mathematics instruction in secondary schools, the *Neue Mathematik* meant an orientation toward learning the modern mathematical language, primarily through frequent repetition and the resulting familiarization with the abstract structures of modern algebra and set theory.

The growing distancing from elite support and the turn toward individual holistic support, especially in social and socio-scientific science, at the end of the 1960s contradicted the ideas of the New Math. The orientation toward the axiomatic deductive doctrine and method of programmed teaching also affected school notions of mathematical giftedness. Thus, the effective acquisition of abstract-formal concepts of algebra and structural mathematics and their manipulation now became a goal of high school mathematics instruction and thus the basis of performance assessment of above-average academic achievement. In contrast to problem solving, this was more a matter of acquiring skills in the mathematical language, the practice of which also required repetition and diligence.

Also in university mathematics, in the beginners' courses, the acquisition of the language of modern algebra and analytical methods were in the foreground and shaped the image of the theoretical mathematician researching in modern areas. The axiomatic teaching method in mathematics classes, which corresponded to modern mathematics and was strongly propagated in the context of the New Math reform, supported a conception of mathematical giftedness that was partly identical with logical, rational intelligence. The growing role of mathematics as a selective subject is reflected also in the large proportions of questions on logical reasoning and geometric-structural ideas in intelligence and admissions tests. The latter was also true throughout Western Europe. Whether and to what extent the opening of the concept of intelligence in the 1980s (e.g. Gardner & Hatch, 1989) had an influence on the conceptions of mathematical giftedness in Western Germany is difficult to prove. In Schefer's empirical study (Schefer, 1969) of the social image of the *Gymnasium* teacher, there are indications as to why the 1968 movement, the ruptures in the teacher education due to the *Bildungsnotstand* (educational emergency), and the critical confrontations with the New Math movement and its "failure" had little impact on the self-image of Gymnasium mathematics teachers and their conceptions of the gifted mathematician. The prevailing notion among university mathematicians about how to narrow the gap between backward and elementary school mathematics and modern conceptual research mathematics consisted of close connections (journals, seminars, personal contacts) of the *Gymnasium* teachers to the universities and their continuous training (Volkert, 2016). This led, among other things, to a continuous modernization of mathematics teaching in the Gymnasium and thus to well-prepared university entrants.

In 1970, the *Bundeswettbewerb* (Federal Mathematics Competition) was established in a West German nationwide mathematics competition consisting of two homework rounds and a mathematical discussion with experts. The structure of the competition hardly involved the respective mathematics teachers. Due to the extracurricular nature of the competition, social factors also had an influence on the success of the students. In 1980, a mathematical students journal "Monoid" was published but its distribution remained at the local level. The "Talent Promotion Mathematics" (two professors from the Department of Mathematics and two from the Department of Educational Science at the University of Hamburg), founded in 1983, was supported by the Federal Ministry of Education and Science (BMBW) as a research project of mathematical talent promotion. In the 1980s, other centers for the promotion of mathematical giftedness were established, most of them affiliated with universities, but they also operated largely locally and without larger networks.

Promotion of mathematical giftedness in East Germany

After the end of the Second World War, in the then Soviet zone and from 1949 in the GDR a unified School system was introduced. The situation among teachers, mainly as a result of denazification of higher educational institutions, was characterized by a shortage of teachers and a resulting focus on methodology for enormously heterogeneous groups. The latter were initially taught largely by lateral entrants. The problem of the social injustice of state support for the elites was less prevalent in the GDR, since children from working-class and peasant families were institutionally supported and children from educated middle-class homes were more subject to quota regulations and thus, in part, to restrictions on access. Since the end of the 1950s, special mathematics and science classes were set up at large universities in the GDR, following the example of the Soviet Union (Kramer & Warmuth, 2007). These classes were open to students who were recommended by their mathematics teachers, took aptitude tests, and later with the introduction of the mathematical Olympics won local and international competitions.

In the early 1960s, the first school, district, and GDR Olympics (*Mathematikolympiaden*) were held (Gräbe, 2005). Mathematics teachers were heavily involved in the competitions, as they were responsible for organizing and correcting the various levels. The international mathematics Olympiads, which had been held since 1959 and in which the GDR participated from the very beginning, established an international performance reference and organizational structures that also applied to competitive sports.

Participation as a team in these competitions, joint preparation, and learning together in special math classes enabled not only athletic competition but also the development of a sense of community that was otherwise not common in the early study of mathematics. The teaching of mathematics in the special math classes attached to the universities was mainly carried out by university teachers from the respective universities and colleges. The development of own special math *matura* and teaching materials as well as numerous translations of Soviet 'student literature' led to broad discourses on how mathematically gifted students could be supported. A network for the promotion of gifted students, which was implemented locally through *Arbeitsgemeinschaften* (study groups), but also nationwide through competitions and math camps, as well as student school correspondence circles and mathematical school magazines ("*Alpha*", "*Die Wurzel*") aimed at promoting mathematical giftedness at both the grassroots and the top levels. General promotion of mathematical interest (*Breitenförderung*) was supported e.g. through the cultural history of mathematics as a compulsory subject in teacher education, interdisciplinary popular science contributions in newspapers and mathematical school journals, or approaches via art and games, (e.g. chess) as well as institutional support of mathematics as a competitive sport and as a tool in technology and science, see also (Weiss, 2017). The broad Soviet research on mathematical giftedness represented primarily through Krutezkii (1976) was known in the GDR. Developmental research on alternative mathematical conceptualizations, tested in special classes, provided a theoretical, empirically-based foundation for the design of models for gifted education, methodologies for their implementation, and the development of curricula, teaching materials, and popular mathematical literature. Both school mathematics teachers and university lecturers were involved in the development of materials.[16]

Promotion of mathematical giftedness in reunited Germany

With the fall of the Berlin Wall, the special math classes at the East German universities were dissolved and the special schools were transformed into *Gymnasium*s with a mathematics and science profile, as they did not fit into the school system of West Germany (Strunck, 2008). The lack of federal political support and thus of funding affected all central "GDR projects" of mathematical giftedness support in the 1990s.

[16] For a detailed comparison of the educational systems in West and East Germany, see the proceedings of the double conference on the joint reappraisal of a separate history by (Henning & Bender, 2003).

The dynamization of giftedness support and thus its promotion into the educational policy of the *Alten Bundesländer* (Old Federal States) was intensively initiated by interest groups through parents' associations. The point of view that these initiatives fought against included the view that especially gifted children did not need any further support. Especially in the Old Federal States, on the one hand, the concept of giftedness was associated with high social esteem; on the other hand, the expenditure of resources and attention on those already "endowed with gifts" was seen as particularly unfair. Possibly because of the high expectations of the public regarding giftedness that are particularly widespread in Germany, teachers' perceptions of giftedness proved to be lower than in other countries. For example, Dahme (1996) found that German secondary school teachers estimate the percentage of gifted students at 3.5%, while American teachers estimate the proportion at 6.4% and Indonesian ones saw it at 17.6%. The conflict of the eligibility of gifted children was resolved primarily by placing the problem within the general framework of inclusive education:

> The support for educating the gifted is booming in many European countries. The generally held opinion during the previous century was that highly able students did not need special attention or extra facilities. Consequently, the task of developing educational and other provisions for the gifted in schools was completely neglected. Only within the past couple of decades has it become more widely recognized and accepted that all children need support that is adjusted to their level of ability, whether low or high, in order to develop their potential to the fullest. (Möncks, 2005, p. 3)

Since the beginning of the 21st century, the codification of gifted education as an original educational and individual right has created a legal basis in the individual German states that both enables and facilitates the promotion of giftedness in schools. This basis includes: legal foundations for skipping grades, shortening of (grammar school) courses of education, opening up the provisions on compulsory schooling, early enrollment, requirements for individual differentiation and individual documentation of learning developments, cooperation with extracurricular institutions, opening up of school district boundaries or permission to take up early studies during the regular school period, as well as improved diagnostics, teacher training and continuing education, counseling for parents and students, differentiated school offerings, and cooperation with universities and technical colleges (Preuß, 2012, p. 54).

Concepts of mathematical giftedness and Interest groups

In the 1980s, Ernest (Ernest, 1991) further developed the concept of interest groups, modeled on British society in sociopolitical terms, by linking the groups to different currents of mathematics philosophy. The characteristics by which different interests in mathematics education are considered include attitudes toward mathematical giftedness and its promotion. The characteristics described there can also be found in different developmental periods of German society and can be justified there by different economic and cultural-historical framework conditions.

The educated bourgeois concept of giftedness of the Enlightenment described in this article, which sees talent as potential and predisposition, shows characteristics of the interest group of the *old humanists*.

The concept of giftedness that is oriented toward performance and achievement, which emerged at the turn of the last century with industrialization, the development of applied mathematics, the need for technical and scientific expertise can be found among the interest group of *technological pragmatists*. Also the achievement-oriented concept of giftedness of the scientific and technical elites of Nazi Germany can be assigned to the *technological pragmatists*.

The concept of mathematics as a subject for selection on the basis of fair comparability, using e.g. mathematical and psychological test, e.g. in the Weimar Republic, as well as the idea of mathematics as a "school of character" in Nazi Germany, relates to ideas of the interest group of *industrial trainers*. Their concept of learning by repeating and training of mathematical skills can be found also in the teaching machines and the programmed learning during the *New Math* movement and in some current e-learning concepts.

Social justice in the promotion of mathematical giftedness is one of the main issues of the interest group of *public educators*. The lack of institutional promotion of mathematical giftedness in the 50s-70s in West Germany as well as the institutional promotion supported by the state in the GDR reflect this perspective.

The concepts of the interest group *progressive educators* are based on constructivist theories and the individual promotion of multiple intelligences, creativity and self-esteem. This idea one can find in pedagogical concepts in the post-war period in West Germany, primary education at the end of the 20th century and today's individual promotion concepts, as applied e.g. in *Waldorfschulen*. These different conceptions of giftedness and its promotion, as well as new concepts due, for example, to the economization of the educational system and the globalization of the economy, determine today's understanding of mathematical giftedness in schools. Here, a reflective approach to underlying interests can contribute both to individual empowerment and to strengthening the common good.

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