Felix Klein (1849-1925): Symbiosis of History, Teaching, and Culture

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Abstract

Based on recent studies, it is possible to explain the hitherto little-noticed link between the fields of history, teaching and culture in Klein's teaching and research projects and the international impact associated with them. This symbiosis can be seen as part of Klein's goal to reform mathematics education and bring mathematics into the consciousness of broad circles as a necessary component of culture.

Klein and Culture – Theoretical Projects

In 1988, Alan J. Bishop (Monash University, Australia) had published *A Cultural Perspective on Mathematics Education*, and the Felix Klein Award for 2015 was given to him. In 2016, he was invited to Hamburg (ICME Congress) to give a talk. In this talk, Bishop suspected that Klein did not have addressed the relationship between mathematics and culture (Bishop, 2017). I was very surprised (see Tobies 1994; 2008) and decided to start my lecture at the ICHME-7 conference with Klein and culture.[1]

In May 1888, Klein had written to the influential civil servant Friedrich Althoff (1839-1908) of the Prussian Ministry of Culture about his interest in culture: "The most important thing for me is the general cultural interest. It is now necessary to overcome the dichotomy between humanistic and realistic education and to transform it into a higher unity, into true education". [Klein to Althoff, a letter dated May 25, 1888][2] Klein's effort to bring together, to unite, here also concerned the aspect of schooling that he had in mind since his inaugural speech at Erlangen in 1872 (see Tobies, 2021, p. 144).

We can see this endeavor in his theoretical projects. As an aim of the *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* [Encyclopedia of the Mathematical Sciences, Including Their Applications, see *Encyklopädie* 1, pp. 898-1935] was stated "to give an overall picture of the position that mathematics occupies within contemporary culture" ("[…] ein Gesamtbild der Stellung geben, die Mathematik innerhalb der heutigen Cultur einnimmt"). (*Encyklopädie*, vol. I: IX).

This concept of culture was very broad, everything that mankind had produced in the course of its history. Klein followed the historian Paul Hinneberg (1862-

[1] Editorial Note: the editors believe that the position of Bishop requires a longer discussion.

[2] German original of this quotation in (Tobies, 1994, p. 5): "Aber das Wichtigste ist mir das allgemeine *Kulturinteresse*. Es handelt sich in jetziger Zeit darum, den überkommenen Zwiespalt zwischen humanistischer und realistischer Bildung zu überwinden und womöglich in eine höhere Einheit, die wahre Bildung überzuführen."

Bjarnadóttir, K., Furinghetti, F., Karp, A., Prytz, J., Schubring, G., Weiss, Y., & Zender, J. (Eds.) (2023). *"Dig where you stand" 7. Proceedings of the seventh International Conference on the History of Mathematics Education.* Münster: WTM-Verlag.

1934) and his colleague Wilhelm Lexis (1837-1914) in this understanding of the concept (see Lexis, 1906). And Klein prompted the mathematician Aurel Voss (1845-1931), a friend of him to elaborate on the topic of mathematics and culture (see Voss, 1914a). Indeed, culture, as an open, multi-facetted, and integrative concept is perhaps best understood in an open system that avoids strict and explicit demarcations. In these respects, the task of understanding *Kultur* turns out to be closely linked to another task prominent around 1900: that of providing a systematic ordering of the various 'Wissenschaften'. Klein had a very decided view of the place of mathematics in this system. Mathematics as a mediating science, we can say today, as an ordering science. In an April 1896 lecture, he expressed:

> Our science, unlike others, is not based on a single period of human history, but has accompanied the development of culture at all stages. Mathematics is as fused with Greek education as it is with the most modern tasks of engineering. It is not only connected with the advancing natural sciences, but at the same time benefits from the abstract investigations of logicians and philosophers. […][3]

Thus, the encyclopedia was to cover in detail not only all mathematical fields and their applications, but also the history, philosophy, and didactics of mathematics. Klein had already written an outline for a final volume VII in 1896, when the contract for the *Encyklopädie* was signed with the publisher B.G.Teubner. This draft clarifies the concern, even though Volume VII remained unfinished:

A. History, Philosophy, Didactics.
1. *History*, i.e., an overview of the advances that have been made over the century in our knowledge and understanding of previous periods of development.
2. *Logic and Epistemology*:
   a) A critique of the fundamental mathematical concepts and of the mathematical methods of proof. (See IA 1.3; II A1; II A1; III B1)
   b) The applicability of mathematics to physical and psychic quantities.
3. *Psychology:*
   a) The psychological conceptions of numbers, time, and space;
   b) The psychology of mathematical thought: specific differences among individuals and the resulting consequences for didactic issues.
4. *The Calculus of Logic* (symbolic systems of logical operations and their application to mathematics).
5. *The Didactics of Mathematics* at educational institutions of all levels and purposes, in Germany and abroad.

B. General overview of the development of mathematical sciences in the nineteenth century.[4]

[3] „Unsere Wissenschaft ist, im Gegensatz zu anderen, nicht auf eine einzelne Periode der menschlichen Geschichte gegründet, sondern sie hat die Entwicklung der Cultur auf allen Stufen begleitet. Die Mathematik ist mit der griechischen Bildung ebenso verwachsen, wie mit den modernsten Aufgaben des Ingenieurbetriebes. Sie reicht nicht nur den vorwärtsschreitenden Naturwissenschaften die Hand, sondern sie partizipiert gleichzeitig an den abstrakten Untersuchungen der Logiker und Philosophen. […] Wir nehmen damit innerhalb der heutigen Gegensätze eine wichtige vermittelnde Stellung ein." (in Klein/Riecke 1900, p. 228)

[4] [AdW Wien] I 170 (Math. Encyklopädie); quoted in (Tobies, 2021, p. 474) (The German original was first published in (Tobies, 1994, pp. 56–57)).

Thus Klein also pursued other large encyclopedic projects and the classification of mathematics in them. In 1906, when the first volume of Paul Hinnebergs *Die Kultur der Gegenwart* [The culture of the Present] appeared, Klein expressed his dissatisfaction with the overall plan and felt as though he had to intervene:

> It must be called a misconception when, in numerous classifications of the sciences (thus also, for example, in the new encyclopedic work "The Culture of the Present"), mathematics is essentially thrown together with the natural sciences. […] At secondary schools, mathematics and the natural sciences are indeed natural allies and are set apart from the other subjects. Yet the science of mathematics has a great deal of significance even independent of every other field of human knowledge; it has relations on the widest variety of fronts and, viewed philosophically, it is not at all connected to any of the natural sciences: in itself, mathematics is a pure human science [*Geisteswissenschaft*].[5]

In 1908 Klein achieved that mathematics, science and technology were integrated into the project *Die Kultur der Gegenwart*; mathematics received, however, its own volume. He headed the mathematics volume (Klein, 1912–1914, see also Tobies, 2008), won friendly authors to present the culture, history and teaching of mathematics in a closely linked way.

The Danish mathematician H.G. Zeuthen (1839-1920) wrote about mathematics in antiquity and the Middle Ages (see Fig. 1). Paul Stäckel (1862-1919) took over the part about early modern mathematics and Klein himself about 19th century mathematics. Aurel Voss wrote about the relations of mathematics to general culture (mentioned above) and about mathematics and philosophy (published in 1914). Heinrich E. Timerding (1873-1945) composed the part on mathematics education (in 1914).

Stäckel could not complete his part. Felix Klein's part became an independent work: his two-volume *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert* [Lectures on the Development of Mathematics in the 19th Century], edited from his Nachlass. In 1926, the editors Richard Courant (1888-1972) and Otto Neugebauer (1899-1990) wrote in the preface to vol. 1: "The occasion for Klein's lectures on the history of mathematics was the plan to prepare a major publication as part of the project *Die Kultur der Gegenwart* [The Culture of the Present]." This statement remained largely unknown internationally because the translation of this preface was not included in the English edition.

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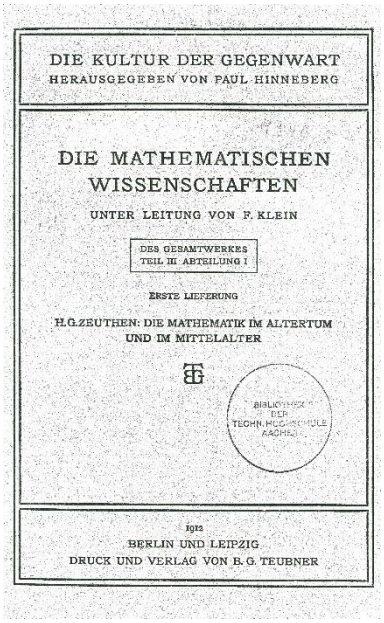
[5] Quoted from Tobies, 2021, p. 475. German original „Man muß es als einen Mißgriff bezeichnen, wenn bei zahlreichen Klassifikationen der Wissenschaften, so z.B. auch in dem neuen enzyklopädischen Werke „Kultur der Gegenwart" die Mathematik prinzipiell mit den Naturwissenschaften zusammengeworfen wird. […] An den höheren Schulen sind ja Mathematik und Naturwissenschaften gegenüber den sonstigen Fächern natürliche Bundesgenossen. Freilich hat die mathematische Wissenschaft auch unabhängig von jedem anderen Gebiet der menschlichen Erkenntnis ihre gute Bedeutung; sie hat nach den verschiedensten Seiten Beziehungen und ist rein philosophisch betrachtet durchaus nicht an irgend eine der Naturwissenschaften gebunden: die Mathematik ist an sich eine reine Geisteswissenschaft. (Tobies, 1994, p. 9)


DIE KULTUR DER GEGENWART
HERAUSGEGEBEN VON PAUL HINNEBERG

DIE MATHEMATISCHEN WISSENSCHAFTEN

UNTER LEITUNG VON F. KLEIN

DES GESAMTWERKES
TEIL III: ABTEILUNG 1

ERSTE LIEFERUNG

H. G. ZEUTHEN: DIE MATHEMATIK IM ALTERTUM UND IM MITTELALTER

1912
BERLIN UND LEIPZIG
DRUCK UND VERLAG VON B. G. TEUBNER

Fig. 1. Title page of the first issue of the mathematics volume of the encyclopedia *Die Kultur der Gegenwart* [The Culture of the Present]

There was a close connection between the aforementioned *Encyklopädie* volume VII, the project *Die Kultur der Gegenwart* and the treatises on mathematical education that Klein had initiated since 1908 as a president of the ICMI. A report on the *Encyklopädie* meeting in Munich in 1910 states:

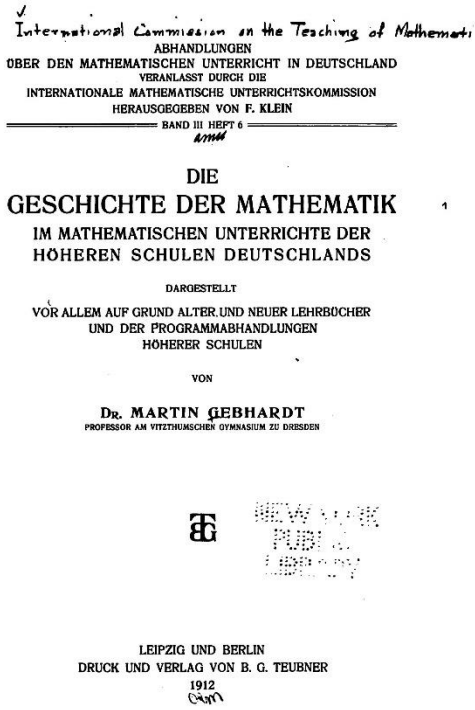
> Regarding volume VII, the committee approves the statement from Mr. F. Klein that, on the one hand, the work of the International Commission on Mathematical Instruction and, on the other hand, the essays planned for the project devoted to the Culture of the Present on the psychological and epistemological foundations of mathematics and on the history of mathematics could serve as preparations for this volume (VII). (German original in (Tobies, 1994, p. 66, Engl. in Tobies, 2021, p. 475))

Klein and the History of Mathematics in the Classroom

Klein considered history of mathematics as an essential aspect of mathematical education. His statements/articles in the context of the educational reform around 1900 reveal his view: History of mathematics should be a key stone of higher education.

A contribution on the history of mathematics in the classroom was included in the *Abhandlungen über den Mathematischen Unterricht in Deutschland* [Treatises on mathematical education], which Klein edited (see Fig. 2). The author Martin Gebhardt (1868-1946) had studied in Leipzig, received his doctorate under Carl Neumann (1832-1925), and became a Secondary School professor in Dresden.

Gebhardt analyzed school textbooks and program writings of teachers on the subject of the history of mathematics in the classroom. The resulting report was part of the works produced by the German sub-commission of IMUK.


International Commission on the Teaching of Mathemati

ABHANDLUNGEN
ÜBER DEN MATHEMATISCHEN UNTERRICHT IN DEUTSCHLAND
VERANLASST DURCH DIE
INTERNATIONALE MATHEMATISCHE UNTERRICHTSKOMMISSION
HERAUSGEGEBEN VON F. KLEIN

BAND III HEFT 6

DIE
GESCHICHTE DER MATHEMATIK
IM MATHEMATISCHEN UNTERRICHTE DER
HÖHEREN SCHULEN DEUTSCHLANDS

DARGESTELLT
VOR ALLEM AUF GRUND ALTER UND NEUER LEHRBÜCHER
UND DER PROGRAMMABHANDLUNGEN
HÖHERER SCHULEN

VON

DR. MARTIN GEBHARDT
PROFESSOR AM VITZTHUMSCHEN GYMNASIUM ZU DRESDEN

LEIPZIG UND BERLIN
DRUCK UND VERLAG VON B. G. TEUBNER
1912

Fig. 2. Title page of volume III, issue 6 of the project *Abhandlungen*…

Felix Klein was already seriously engaged in approaching historical accounts as a *Privatdozent* (from January 1871 to summer 1872 in Göttingen). He had been influenced by Leopold von Ranke's (1795-1886) historicism, a source-based historiography. Later Klein stressed: "My way of doing history. Posting specific questions, then comparing the sources." How much Klein valued sources can be seen in his editions of works by mathematicians (Plücker, Möbius, Gauss) and his commitment to the collected works of other mathematicians (Clebsch, Grassmann, Riemann). See in detail (Tobies, 2021, pp. 477–81).

In order to promote general mathematical literacy, Klein recommended that teacher candidates should study classical sources (Euclid, Archimedes, Apollonius, and so on). He acquired mathematical-historical literature for the so-called *Lesezimmer* (the reading room, a work room for students). (For the student's *Lesezimmer*, initiated by Klein at the universities of Leipzig and Göttingen, see Tobies, 2021, pp. 218–21)

Klein integrated historical themes into in-service training courses for secondary school teachers. His visits to school classes in 1893 had taught him the importance of such topics for teaching mathematics. The topic chosen for 1894 (the three classical geometrical problems that cannot be solved with compass and ruler alone) established Klein's expert status for teaching issues, nationally and internationally. He used the opportunity to connect these topics with new insights (the proofs for the transcendence of *e* and π, e.g.). His booklet (Klein 1895, 66 pp.) on these topics was immediately translated into French, Italian, English, Japanese, and Russian by mathematicians who also became driving forces behind the reform of mathematics education.

Moreover, Klein inspired Conrad H. Müller (1878-1953) to practice the new type of historiography. Müller completed his doctoral degree under Klein with a dissertation on *Studies on the History of Mathematics, Especially of Mathematical Instruction at the University of Göttingen in the 18th Century,* in 1903. Three years later, Müller planned his *Habilitation,* which was supervised by Klein. It has been overlooked until now that this would result in the first *venia legendi* awarded in Göttingen for "mathematics, in particular the history of mathematics." (Tobies, 2021, p. 479) Müller taught history of mathematics in Göttingen and later also in Hanover.

Klein used some of his research seminars to study historical sources in order to introduce new subject areas to prospective teachers. Above all this included differential and integral calculus, whose founders are considered to be Newton and Leibniz, although there were numerous predecessors. By analyzing the sources, Klein intended to show that this mathematical knowledge, which had existed for more than 200 years, should be a significant part of secondary school teaching. This was an important demand of "Klein's instruction reform," but he did not initially succeed in making this demand obligatory in the new (Meran) mathematics curriculum in Prussia. He opened his seminar on the history of infinitesimal mathematics in the winter semester of 1911/12 with the words:

> The purpose of this year's seminar is to study the history of the origin of our present-day infinitesimal mathematics from the sources themselves.
> We start with the theory of infinitely small quantities, as it was handled by Leibniz and his students. We still have to learn the method of series expansions and finally the application of the notion of limit.
> In addition, we will have to investigate how the mathematicians before Newton and Leibniz dealt with the infinitesimal problem: starting from Archimedes up to the more recent ones.
> Klein (November 1, 1911)[6]

In this seminar on the history of infinitesimal mathematics, 23 papers were presented, including by his daughter Elisabeth Klein (1888-1968) and by Iris Runge (1888-1966), who delivered the first lecture and was the only one to take two talks (see Tobies 2012). They were very well prepared to teach the new subject at the

[6] [Protocols], vol. 29, p. 157. - Klein had given only his opening lecture and then went to a sanatorium to recuperate. The rest of the seminar was led by Rudolf Schimmack (1881-1912), who had received the first venia legendi for didactics of mathematics in 1911, promoted by Felix Klein.

secondary schools, which was finally also made obligatory in the mathematics curriculum of the Prussian schools in 1925. Klein could still take note of this, as we can read in the 3rd edition of his *Elementary Mathematics from a Higher Standpoint* (vol. 2).[7]

New examination regulations and new curricula

Within the German Mathematical Society, the idea of new examination regulations for teaching candidates had been a topic of discussion since the 1890s. From his first trip to the United States in 1893, Klein brought back new ideas, which he formulated in a report to the Prussian Ministry of Culture. In short, Klein proposed: Women should be allowed to study;[8] training of future teachers needs to be changed; financial support is needed (see Tobies, 2021, pp. 410–411).

At the time, modern mathematical and scientific education was also regarded a matter of international competition. In 1896, the Royal Commission on Technical Instruction in London wrote in a report on German higher education:

> They [our foreign rivals] are convinced that the nation which has the best schools is the best prepared for the great industrial warfare which lies before us, and no money appears to be grudged for the creation, equipment, and maintenance of educational institution of all grades and especially of the science laboratories which, as we have seen, are being multiplied in Germany. (Magnus et al., 1896, p. 412)

With regard to applied physics and mathematics, however, this judgment could by no means refer to the German universities. Felix Klein was aware of the demands of German engineers and therefore aimed to achieve more application-oriented teaching at the universities and at secondary schools. On June 2, 1897 he wrote to the aforementioned Althoff – the influential official in the Prussian Ministry of Culture – about his new ideas for the examination regulations. The main idea was a "new *Facultas* for applied mathematics" for secondary school teachers. This idea was stimulated not only by the complaints of engineers, but also by the vision that the Ministry would thereby support appropriate teaching positions at all universities and finally open the way for the development of research in applied mathematics.

At the University of Göttingen, Klein was able to raise funding from industrialists and engineers, who in turn expected to benefit from new research results and from the improvement of general education. On February 28, 1898, Klein founded the *Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik* (Göttingen Association for the Promotion of Applied Physics and Mathematics).[9] One of its founding members, the engineer and industrialist Anton von Rieppel (1852-1926) even described the improvement of pedagogical training as the

[7] See Klein 2016 [^{3}1925], p. 302.

[8] Klein achieved this from the autumn of 1893 (see Tobies, 2022).

[9] The extension to mathematics did not occur until December 17, 1900, see (Tobies, 2021, pp. 439–445).

decisive motivation behind the foundation of the Association (see Tobies, 2021, p. 444).

Most teacher candidates chose three subjects for the teaching examination. Now there was the possibility to acquire the teaching qualification (*Facultas*) for pure mathematics, applied mathematics and physics. By the year 1910, 178 students in Prussia had earned the new qualification to teach applied mathematics. Most of the latter went on to teach at secondary schools. The selection of application areas – recorded in the examination regulations – was expanded in the following years (see Table 1).

Table 1. Applied Mathematics in the Prussian examination Regulations for Teaching Candidates at Secondary Schools

1898	1917	1921
1. *Descriptive geometry* up to the theory of central projection and skills in graphical methods. 2. *Technical Mechanics*: Mathematical methods, esp. graphical statics. 3. Basic *geodesy* and elements of advanced geodesy alongside the theory of the adjustment of observation errors.	*Mastery of graphical and numerical methods* (descriptive geometry, graphical calculation, approximate calculation) and their application to at least one of the following fields: 1. *Astronomy* 2. *Geodesy* 3. *Meteorology and geophysics* 4. *Applied mechanics* 5. *Applied physics* 6. *Mathematical statistics and actuarial mathematics*	Familiarity with the aspects of analysis most important to applications – particularly its *computational, graphical, and instrumental methods* – with descriptive geometry, mechanics (including graphical statics & kinematics), probability and approximation theory. More in-depth theoretical and practical studies in at least one of the following areas of application: 1. *Astronomy*, 2. *Surveying*, 3. *Meteorology and geophysics*, 4. *Applied mechanics*, 5. *Applied physics*, 6. *Financial mathematics, mathematical statistics, and actuarial mathematics*, 7. *Technical sciences* (*e.g., electrical engineering, heat engineering, aeronautical engineering, or the statics of structures*).

In the advanced training course, during the Easter Vacation of 1900, the secondary school teachers already in practice were also made familiar with the importance of applications of mathematics to teaching. In order to reach an even wider circle of teachers, Felix Klein and the physicist Eduard Riecke (1845-1915) collected the pertinent lecture notes, and published them in a book *Über angewandte Mathematik und Physik in ihrer Bedeutung für den Unterricht an den höheren Schulen* [On applied mathematics and physics in their importance for teaching in secondary schools] (Klein/Riecke, 1900). This book was quickly noticed and reviewed abroad as well.

Klein's commitment to the improvement of the mathematical instruction included education at all levels. In a parliamentary speech on April 7, 1911, he expressed: "[…] the ideal that I have in mind is that we should regard education as a

large whole, beginning with kindergarten (with its own particular and highly interesting problems) and extending all the way up to higher education and far beyond." (Quoted in (Tobies, 2021, p. 508, German original Tobies, 2019, p. 437))

Klein endeavored in numerous committees to achieve this ideal. Especially, he was involved in three important education committees nearly simultaneously: a German Committee for Mathematical and Natural-Scientific Instruction (*Deutscher Ausschuss für den mathematischen und naturwissenschaftlichen Unterricht*) created in September 1907 to put the reform into practice; the aforementioned ICMI, and a third committee in the Upper House (*Herrenhaus*) of the Prussian Parliament, after the University of Göttingen had elected him, on December 14, 1907, as its (non-party) representative there. Klein's parliamentary position came to the end only in 1918 with the downfall of the German Empire.

Klein early reflected that active countermeasures needed to be taken to curb the surging nationalism of the post-Bismarck era:

> Particularly in our time, when national chauvinism is at its orgiastic peak, it must be close to the hearts of all truly educated people to counteract a movement that is increasingly intent on alienating even the best citizens of different nations from one another. It is as though the whole human spirit is vanishing along with the humanism that was once so rightly celebrated. […] Intellectual exchange between modern cultures does not entail the elimination of national differences; rather, it leads to a clearer understanding of their true character and values, and this higher knowledge creates a friendly relationship among nations.[10]

Klein had written these words on March 11, 1908 as part of a draft proposal for establishing a Göttingen Institute for Foreigners (*Göttinger Institut für Ausländer*), which he sent to the Ministry of Culture in Berlin.

As a member of the parliament, Klein supported Carl Heinrich Becker (1876-1933), a (politically independent) adviser in the Prussian Ministry of Culture.[11] In 1916, Becker wrote a "Memorandum on the Future Expansion of Foreign Studies at Prussian Universities" (*Denkschrift über den künftigen Ausbau der Auslandsstudien an den preußischen Universitäten*). Both were completely in favor of promoting a better understanding of other cultures, with the justification that such knowledge might make future conflicts avoidable.

Klein composed a twelve-page text titled "Bemerkungen über die Aufgaben der Universitäten nach dem Kriege, insbesondere in der Richtung auf ein verbessertes Studium der Verhältnisse des Auslandes" [Remarks on the Objectives of Universities After the War, Particularly Towards an Improved Study of the Situation in Foreign Countries]. He formulated specific suggestions for modern language studies, which he enumerated in a parliamentary speech on July 8, 1916:

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[10] [UBG] Cod. MS. Math. Arch. 50[21], fols. 54–57v, at 56–56v (Klein, a letter to the Prussian Ministry of Culture, dated March 11, 1908), quoted in Tobies 2021: p. 511 (German original in Tobies, 2019, p. 438).

[11] Becker was an orientalist. In 1921 and 1925–1930, he served as Minister for Culture in Prussia (independent), a reformer of the system of higher education in the Weimar Republic.

"Besides French and English, additional foreign languages should be taught, especially Russian, Polish, and Italian; but Hungarian, Scandinavian, Dutch, and oriental philology should also be established at universities. Education in modern languages should not be elective but rather obligatory […]." (Quoted in (Tobies, 2021, p. 532; German original in Tobies, 2019, p. 465–66))

Klein had a broad visionary understanding of education and culture. In 1918, he discussed further problems with Becker: the unity of pure and applied research; the full establishment of newer areas such as mathematical statistics, photochemistry, radiology, but also history of mathematics, and the didactics of the exact disciplines; the creation of professorships in the field of education, etc. (see Tobies 2021: p. 534).

Concluding remarks

Klein's intention to make mathematics effective for the general public led to a symbiosis of culture, history of mathematics and teaching. This path was accompanied by theoretical projects and practical measures:

The inclusion of mathematics (plus its teaching and history) in publication projects. Klein recognized the opportunity to make the importance of mathematics for general education and culture accessible to wider circles. He prepared a volume of *History, Philosophy, Didactics* as part of his *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* [Encyclopedia of the Mathematical Sciences Including Their Applications]. Klein edited a mathematics volume (1912-14) as a part of the project *Die Kultur der Gegenwart* [Culture of the Present]. At the same time, he managed *Abhandlungen über den mathematischen Unterricht* [Treatises on mathematical education] (5 vols., 1909–1916). This work is reflected in Klein's research-oriented teaching.

The establishment of regular holiday courses (= further education courses for teachers who were working at secondary schools), for the first time in 1892 (in 1909 for the first time for female teachers). In 1893, Klein himself had visited some secondary school classes and examined which topics were valuable for further education. The courses were held every two years. Only the First World War caused a break. At the new start in the Weimar Republic, the representative of the Prussian Ministry of Education promised to continue in the spirit of Klein and to convey to the youth the eminent cultural significance of mathematics and its applications.

A new examination regulation for math teacher candidates in 1898, which for the first time included a teaching qualification in applied mathematics. Klein had achieved this through the support of the Prussian Ministry of Culture and that of industrialists with an interest in science. The examination regulations were the basis for new lectures and seminars, in which Klein also integrated the history of mathematics.

We see Klein's recognition as an expert in mathematics education in his membership of the *Comité de Patronage* of the journal *L'Enseignement mathématique,* as well as in his positions in the well-known reform bodies both in Germany and

internationally[12]. It is less well known that Klein, as the "Foreign Minister of Mathematics in Germany" (A. Fraenkel 1967/2016), fought tenaciously after the First World War to ensure that the main aspects of the Meran Reform were incorporated into a new Richert Reform (1925) in Prussia (see Tobies, 2021, pp. 551–556). All his efforts show a close symbiosis of culture, history and teaching of mathematics.

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[12] On Klein's ICMI presidency, see also the recent book (Furinghetti & Giacardi, 2022).

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