Hebrew authors in the 12th-16th centuries on teaching mathematics

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Abstract

During the last decade, many works in Hebrew (manuscripts and printed books) from the 12th–16th centuries on scientific and especially mathematical topics have been deciphered. In these works, one can find mainly topics like arithmetic and geometry. Apart from the mathematical content, however, we can sometimes find (especially in each introduction but not only there), more information about other topics. Below are presented examples of the diverse information that can be found in the introductions to mathematical works written in Hebrew during the 12th–16th centuries, with a special focus on information that allows us to learn about the didactic views of the authors of these works. As research in teaching in general and research in mathematics education are relatively new research disciplines, these views are particularly interesting. Here we find evidence of didactic thinking long before the subject of mathematics teaching became a canonical university discipline. Scholars and students studying mathematics education will be interested in discovering that didactic issues in mathematics teaching are as old as mathematics itself. Special attention will be paid to Elijah Mizrahi's Book of the Number (posthumous print in 1533), a unique figure among the authors of mathematical books in Hebrew from the 12th–16th centuries.

Keywords: Hebrew mathematicians, mathematics education, Middle Ages.

Introduction

The Hebrew mathematical literature (mathematical texts written in Hebrew) began to develop in Jewish communities in Europe in the late 11th and early 12th centuries. Initially, it emerged in Spain and southern France, but later mathematical texts were also written in Italy and Judeo-Byzantine communities (Lévy, 2011).

During the Middle Ages, the educational frameworks in Jewish communities differed from those in Christian communities in Europe. In Jewish communities, learning started at an early age, typically between 5 and 7 years old, and was conducted under the guidance of the local rabbi. The initial stages of education emphasized basic skills such as reading, writing, and basic arithmetic. However, the primary focus of education was centered on religious texts. Jewish communities varied in their attitudes toward secular sciences, including mathematics. Questions such as what the place of mathematics in the context of the community culture should be, what are the relations (if any) between mathematics and Jewish Law or rabbinical literature, and what specific subjects should be taught, were asked and the answers varied between and within communities. Despite these differences, strong connections existed between the communities, and knowledge transfer (such as through manuscripts and letters) also occurred (Freudenthal, 2011; Lévy, 2011; Aberbach, 2002).

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In the 11th-12th century, in Barcelona, Abraham bar Hiyya (ca. 1065-1145) was probably among the first to write mathematics in Hebrew. His two books, a scientific encyclopedia, and a work on practical geometry, are based on Arabic sources (Bar Hiyya, 1912, 1952). In these texts, Bar Hiyya created an entirely new mathematical language in Hebrew, which was one of his greatest achievements. A few decades later, Abraham Ibn Ezra (ca. 1090-1167) wrote "The Book of Numbers" in addition to other texts on various subjects such as poetry, biblical commentary, grammar, and astronomy. His book on arithmetic had a significant influence on all his successors in this field.

Over the course of around 100 years, a large translation project took place in the south of France, during which the major works of Greek mathematics were translated from Arabic into Hebrew, along with their commentaries and adaptations. Levi ben Gershom (Gersonides) (1288-1344) and Immanuel Bonfils (1340-1365) are two important figures who contributed original works mainly in mathematics, but not exclusively.

In the 15th century, Hebrew mathematicians in Italy, such as Mordecai Finzi (d. 1475), were not only influenced by Arabic literature but also by Latin literature in the field. Some of this mathematical literature also made its way eastward to the Byzantine Jewish communities, where figures such as Mordecai Comtino, Caleb Afandopoulou, and Elijah Mizrahi worked and wrote mathematical texts, as well as texts on other subjects (Lévy, 2011).

The topics covered in Hebrew mathematical texts are highly diverse, including a wide range of subjects such as Euclidean geometry, spherical geometry, arithmetic, number theory, algebra, and more.

Research Objectives and Methodology

This research will deal with Hebrew manuscripts with arithmetical content from the 12th to the 16th centuries. To date, only partial research has been conducted on these manuscripts. In the 20th century, a limited number of critical editions were prepared (Ibn ʿEzra, 1895; Levi ben Gershon, 1901; Bar Hiyya, 1912, 1952) and some manuscripts received attention through various articles written about them (Wertheim, 1896; Schub, 1932; Levey, 1952, …). Moritz Steinschneider (1816–1907), a German bibliographer and scholar made significant contributions to the study of Jewish literature, including Hebrew manuscripts. Steinschneider's research focused on the identification, description, and analysis of Hebrew manuscripts from the Middle Ages and Renaissance periods (Steinschneider, 1964).

In recent decades, the subject has gained renewed interest. Several doctoral theses were devoted to critical editions of mathematical manuscripts (Wartenberg, 2007; Segev, 2010; Aradi, 2015). Scholars like Tony Levy (2011), Gad Freudenthal (1993, 2011), and others have published articles examining the mathematical content found in these manuscripts, as well as broader aspects of Jewish scientific activity during that era. In addition, the translations, and annotated transcripts of

almost all surviving Hebrew medieval manuscripts on arithmetic have recently been uploaded on Mispar website[1].

These texts cannot be named textbooks in the modern sense[2]. While they were created for study purposes, their primary audience comprised individuals with a specific interest in mathematics. Despite not being utilized in a formal educational setting with teachers, classes, and a structured curriculum, it is possible to identify within them passages that reflect didactic concepts of the authors. However, it is worth noting that, to the best of my knowledge, there has been a lack of attention given to the didactic aspects specifically within the context of mathematical manuscripts.

Freudenthal (2011) talks about the change in the agenda of the history of science and Jewish studies: a shift from a positivist view (which is mainly interested in the contributions to science) to a broader view which is interested also in cultural and sociological contexts: beliefs of scientists or the sociological background in which they worked. We can also add to this discussion the history of mathematical education. Also, in this field, it is important to know not only what and how mathematics was taught in a certain period, but to understand the cultural-sociological background and the mathematicians' opinions on the teaching of this subject.

In the Hebrew manuscripts from the 12-16th centuries, one can find mainly topics like arithmetic and geometry. But apart from the mathematical content, we sometimes find (especially in their introductions but not only there) references to more general subjects such as:

- Details about the author's life and his social and cultural environment.
- Philosophical discussions about sciences in general and the place of mathematics among them.
- Reasons for writing a mathematics textbook, why study mathematics?
- The author's opinions on didactic issues (How should mathematics be taught? What is a good mathematics textbook?) The discussion on these issues also reveals the author's views about mathematics in general (is it a theoretical subject or rather a practical one?)

In the following, different answers to the last two questions above will be presented, as they can be found in Hebrew texts. Table 1 shows the list of texts discussed in this paper. These answers were sorted into several categories so that it would be easier to get a coherent picture of the common didactic views on the teaching of mathematics.

[1] Mispar website: https://mispar.ethz.ch/wiki/Main_Pag

[2] See in Selander (1988) and Stray (1994) reference to modern characteristics of a textbook.

Table 1: Hebrew Authors

	Author	Text	Place and Date
1.	Abraham Bar Hiyya	Treatise on Measurement and Calculation	12th century, Spain
2.	Gersonides	Work of Calculation	1321–2 South France
3.	Joseph Ben Moses Sarfati	Ir Sihon	before 1384? Spain
4.	Isaac Ben Solomon Ben Al-Aḥdab	The Epistle of the Number	1429–1433, Sicily
5.	Isaac Ben Moses Eli	The Art of Number	15th century, Spain
6.	Mordecai ben Eliezer Comṭino	The Book of Calculation and Measurement	15th century, Constantinople
7.	Gad Astruk	Laws of Properties	1503, Italy
8.	Elijah Mizrahi	The Book of Number	1534, Constantinople
9.	Anonymous	Sefer Sifra	Portugal 1464 –1471

Some of the Hebrew authors quoted are well-known names like Abraham bar Hiyya (11th century) and Gersonides (14th century), but many others have come to our attention only with the progress of research in recent years.

The English translations of some of the citations below are taken from the Mispar website. In this case, the references are to the pages of the manuscript as they appear on the website.

The reasons and the goals for writing a mathematics textbook; Why study mathematics?

These questions do not deal necessarily with didactic issues, but rather with sociological ones.

We can find several answers:

a. The causes and the goals are practical: we need to know mathematics to solve different life problems. Although this is a very practical goal, it is also related to moral issues: a desire to conduct oneself in the community in a fair manner and to uphold justice.

Abraham bar Hiyya who lived in Barcelona in the 12th century, thought that mathematics was a useful discipline that would help us to keep the Torah's laws, to prevent injustice and robbery.

Bar Hiyya (1065-1136/1145) known also as Savasorda, was a Catalan Jewish mathematician, astronomer, and philosopher. He saw the poor mathematical knowledge of the Jewish community in the south of France and, to help them, he

wrote the "Treatise on Measurement and Calculation" - *Hibbur ha-Meshihah ve-ha-Tishboret* which was translated into Latin by Plato of Tivoli as "Liber embadorum" in 1145.

In his introduction, he explains why he wrote the book.

The Jewish people in the south of France did not speak Arabic (unlike bar Hiyya who was fluent in Arabic) and therefore they could not learn arithmetic and geometry because at that time the available scientific texts were written mainly in Arabic, with a minority of them in Greek or Latin (Freudenthal, 1993). As a result, he says, mistakes were made in measuring the lands and dividing inheritances.

Bar Hiyya's book was one of the first books with mathematical content written in Hebrew and therefore he created an entirely new mathematical terminology in Hebrew (Sarfatti, 1968).

> I have seen that most contemporary scholars in France are not skillful in measuring land and do not divide it cleverly.
> They severely belittle these matters, and divide land between heirs and partners by estimate and exaggeration, and are thus guilty of sin …
> **And because I saw the great damage and loss caused by them**, I bothered myself, and even though I am not fit and worthy to write about the measurement of the land and its division, and bring about it signs and evidence from the wisdom of the measurement and the wisdom of the number, which have no approximation … as I could, **perhaps a person will learn them and remove from these countries this mistake** that is practiced among them.
> Those who wish correctly to learn the ways to measure surfaces and to divide them, must necessarily thoroughly understand the general theorems of geometry and arithmetic, on which the teaching of measurement ... rests. If he has completely mastered these ideas, he ... can never deviate from the truth. (Bar Hiyya, 1912, pp. 2, 5,6)[3]

About 300 years later, in Constantinople, Mordecai Comṭino (1402-1482, f. 26r) in his "Book of Calculation and Measurement" gives similar reasons:

> For, once the two sides compromise, concerning an acquisition of a certain amount, on a certain price for its counting portion, … if the calculation fails, due to their lack of knowledge in arithmetic or land measurement, **injustice, and iniquity will be caused from that, and justice will be taken thereof.** Therefore, I decided to compose this book to remove injustice from human beings and so justice will prevail among them.

Gad Astruk (1503, f. 191v) in the introduction of his "Laws of Properties" writes also about the need to learn arithmetic:

> It is known to those who have eyes to see, and ears to hear, and a heart to know, **that man has no ability to make a true judgment and to pronounce truth, unless he is skilled in arithmetic**.

Even the name of the book, "Laws of Properties", emphasizes his practical goal**.**

[3] The emphasis in the text, in this quote as well as in the ones that follow it, are not in the original, their purpose is to emphasize the sections relevant to the discussion.

b. The reasons and the goals are related to the inner world of the author – his interest and a strong desire to learn mathematics. These are sometimes joined by the desire to help others who are also interested in the subject.

Joseph ben Moses Sarfati writes in the introduction of his "Ir Sihon"[4] (before 1384, f. 51r):

> … **my heart and kidneys urged me to write a book about it,** as I extracted from studies of the ancient scholars in a clear language, clarified for every thinker, for its way is hidden and concealed and requires a broad knowledge as the entrance of a hall.
>
> The people who are interested in the words of wisdom in this book will benefit much from its additional interpretation.

In addition, he will benefit from writing: When we write a textbook, we learn by ourselves and understand better. Sarfati (f. 51r) writes:

> I myself will also benefit that the science will be preserved within me properly when actualizing it by the phrasing of my language, and so I will gain learning as well as teaching.

Another example can be found in the prolog of "Sefer Ṣifra" (written in 1537, probably translated and adapted from an earlier text[5]). There the author expresses his strong desire to learn mathematics and in addition complains that he could not find books in Hebrew on the subject (Sefer Sifra, f. 197r).

> This is a mathematics book [called] Ṭeshifra.
>
> Its cause: I myself, already from youth**, my soul desired to acquire wisdom and to seek for arithmetic**, if only I will find them in a book engraved by a pen of a skilled writer. For the reason that this is an external science, which is not found before us in Israel …, my thoughts, therefore, aroused me to collect the sayings from anyone that has them …

c. We learn math because it develops thinking.

This concept of mathematics as a tool to improve thinking has been widely accepted since ancient times (Plato) until today, although it is still controversial.

So, it is not surprising to find it among the Jewish mathematicians as well.

We can see an example in the following quote from the book "Sefer Ṣifra"(f. 197r) mentioned above:

> … the arithmetic problems, the Book of Conics, the numerous techniques for questions on geometry, the measuring of weights, and many like those – the intention in them is to sharpen the mind and train the rational faculty through methods of proof until a person acquires the knowledge of demonstrative reasoning ….

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[4] The name of the work ʿIr Siḥon refers to a biblical phrase: Ir Siḥon = the town of Sihon (king), the name of this town was Heshbon (ḥeshbon = arithmetic).

Book of Numbers 21, 27: בֹּאוּ חֶשְׁבּוֹן; תִּבָּנֶה וְתִכּוֹנָן, עִיר סִיחוֹן = "Come to Ḥeshbon, may it be built and established as the city of Siḥon"

[5] Steinschneider, Moritz (Moshe). 1893–1901. Die Mathematik bei den Juden. Berlin-Leipzig-Frankfurt: Kaufmann; repr. Hildesheim: G. Olms, 1964 and 2001. pp. 216-217 [i478-i479]

d. Because there are no good textbooks for mathematics (in Hebrew), the goal will be to write a better book.

A possible explanation for the lack of good math textbooks can be found in the section above from "Sefer Ṣifra": since mathematics is a secular subject – "*an external science*", there are fewer books in Hebrew on the subject.

> For the reason that this is an external science, which is not found before us in Israel…

But this statement is somewhat puzzling: During this period, the 16th century, quite a lot of math books had already been translated into Hebrew and written in Hebrew. [6]

Many other authors emphasized the fact that the study of mathematics helps to understand the world created by God. We can see for example what Joseph ben Moses Sarfati (f. 51r), mentioned above, writes:

> Since I have seen that arithmetic is a magnificent and extremely lovable science and the Creator, blessed be He, has placed the secret of His unity in the property of number …

The meaning of his words is that arithmetic contains signs of the unity of God.

Many authors wrote about their desire to write a textbook that would be better than the existing ones – a natural and even obvious desire. Later we will try to delve into the issue and characterize exactly what they wanted to improve.

Didactic issues: How should one learn math? How should mathematics be taught? What is a good mathematics textbook?

The discussion on these issues sometimes reveals the author's views about mathematics in general: is it a theoretical subject or rather a practical one?

We can speculate what the answers to the above questions are by examining the structure of the book itself: are proofs provided for all the theorems, where are they located, how many and what types of examples are provided for each operation, is more than one way of performing a calculation operation presented, etc. But these speculations gain more validity when the author himself adds explanations regarding his considerations for choosing a certain way to write the book.

In the Euclidean tradition, the proof of the theorem is given immediately after its formulation. Throughout history, some mathematics textbooks were written so that the theorems and examples were presented first, and the proofs were added in later chapters. Such a structure was probably intended for those who were only

[6] The prologue of "Sefer Sifra", quoted above, appears in only one of the twelve manuscripts that include this text, a manuscript found in Paris. It is possible that this is a late addition (not by the original writer), and it reflects the situation in the Ashkenazi countries where the rabbinical leadership opposed any form of secular studies (Aradi, 2015; Aberbach, 2002).

interested in the practical side of the subject. It also facilitated abridged editions to be written for those who did not wish to delve into the proofs. Al-Khwarazmi's famous essay "The Compendious Book on Calculation by Completion and Balancing" is structured in this way (proofs are grouped in a separate chapter at the end).

When the subject of the study is arithmetic, and the books are mainly intended for traders and other people whose purpose was mainly practical, the second approach (which summarizes and shortens the theoretical explanations) is preferable. And, in the Middle Ages, starting with Fibonacci, many books of this type were written, including Fibonacci's "Book of Calculations"[7] and "Treviso Arithmetic".

The following are the opinions of Jewish mathematicians on these issues:

a. Levy ben Gershom – Gersonides who lived in the south of France in the 14th century, supports learning the theorems with their proofs because this is the way to reach a deep understanding. He writes in his "Work of Calculation" – Sefer *Ma'ase Hoshev* (1321, p.1):

> Hence, it is clear that only with the greatest difficulty can perfection in this science be complete without knowing the reasons. But with knowledge of the reasons, it can easily be complete.

According to this view, his book is written in a Euclidian style, where every theorem is followed by its proof.

b. Isaac Ben Moses 'Eli, in the introduction of his book "The Art of Number" - *Meleket ha-Mispar*, 15th century (f. 7v), takes pride in a concise essay and complains about the waste of time that can be caused when the textbook is not such.

> When I saw the **lengthiness of the discussions of the ancient scholars**, … and that the necessary methods and teachings in astronomy, geometry, … are explained in those books merely in great difficulty; … and **since the teaching that is [based] on brief and comprehensive methods is better chosen for the student as well as for the teacher**, for every lengthiness of words and loquacity are the exhaustion of the body.

c. Mordecai Comtino (1402-1482) follows the same way of writing a short book ("The Book of Calculation and Measurement"), but he adds a new reason (f. 26v):

> By this, I followed the practice of the Christians to speak briefly **so as not to confuse the student.**

d. Gad Astruk (1503, f. 192r)) also wants to write a short but inclusive treatise in contrast to other books – "Laws of Properties". He even mentions the names of other authors who wrote much more in-depth books - probably Abraham Ibn Ezra and Levi Ben Gershom:

> However, the first reason forced me to write a short treatise, in which I include all the matters of numbers in brief.

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[7] Fibonacci himself sometimes referred the readers of the "Book of Calculation" to his other works such as "The Book of Squares" to review the proofs of the theorems contained therein.

> This has brought me in the path of this composition [or translation/ copying] not to brag and boast and take a crown on my head and not to talk on things greater and wondrous than me as did the wise man R. A. [probably R. Abraham Ibn Ezra] and Maestro Leon [called R. Levi ben Gershom] in Maase Ḥoshev, for each of them ascended in the degrees of sciences, grasped the branches of ideas.

e. We can summarize the debate between these two opposite approaches (a detailed book that includes all the theorems and their proofs compared to a concise book that is more convenient for practical use) with the story given in the introduction of "The Epistle of the Number" (*Igeret ha-Mispar*) by Isaac Ben Solomon Ben Al-Aḥdab (1350-1429/1433):

The admirers of an Arab scholar (apparently Ibn al-Bannāʾ) asked him to write a short treatise for them, containing all subjects of the Science of Number. He (the Arab scholar) wrote a very short epistle for them. This epistle was so concise they couldn't understand it.

So, they asked him to write a commentary. The Scholar wrote a very profound and lengthy commentary, intelligible only to very gifted people.

Al-Ahdab, traveling in Arab countries, got to know the epistle and studied it.

When Al-Ahdab arrived in Syracuse he met scholars also interested in arithmetic. They asked him "to compose for them a short but extensive book on this wisdom".

Al-Ahdab decided to translate the short treatise of the Arab scholar and add his explanations.

So, what can the moral be? Perhaps it doesn't matter how long a textbook is, but how clear and well-written it is.

Didactic views of Elijah Mizrahi (15-16th century)

Elijah Mizraḥi (1450-1526) was a prominent personality in the Jewish community in Constantinople. He wrote several treatises on religious subjects (such as a super commentary on Rashi's commentary on the Torah), but he was also interested in scientific subjects. His arithmetic book *Sefer ha-Mispar* ("The Book of Number") is particularly interesting for our discussion. (This is also the first Hebrew book on arithmetic to be printed.)

Sefer ha-Mispar ("The Book of Number") is a large essay of approximately 200 pages: it contains three parts dealing with arithmetic operations on integer numbers, fractions, and sexagesimal fractions. It also includes a chapter on square and cubic roots and arithmetic, geometric and harmonic proportions, and a chapter on 99 arithmetic and geometric problems.

For each of his subjects, Mizrahi first defines and explains the arithmetic operations and related notions, and then describes methods of solution accompanied by examples. He accords special attention to verification techniques. Then, in a separate section at the end of the relevant chapter, Mizrahi explains and proves his methods. This structure of the book certainly facilitated the publication

of the abridged 16th-century edition, Compendium of Arithmetic, which contained only the algorithms presented in the first two sections.

In each of the parts of his book (and not only in the introduction) we can see his great interest in didactic issues.

Mizrahi explains his reasons for writing the book in the introduction. His approach is theoretical, and he emphasizes understanding. He goes against the "ancients" who only wanted to give short methods. Contrary to them, his view is that detailed explanations and proofs are necessary for several reasons:

a. Explanations and proofs are necessary for students to really (deeply) understand the subject being studied.
b. The explanations and proofs are necessary for the development of the learner. A deep understanding of the proofs will help the learner to develop new methods on his own.
c. The explanations and proofs give tools to the learner to solve new problems, different from those presented by the teacher.
d. Teaching multiple methods will help students understand more than studying only one method.

The following excerpts exemplify these claims (Segev, pp. II-4-5) :

> The reasons that urge me to speak, even if the ancient scholars have already preceded me in this, are two:
>
> - The first reason is that **all the ancients**, whose works concerning this science have come down to us, have endeavored in informing the methods and the ways by which one reaches every principle of this science.
>
> Yet, their chief intention was only to summarize the methods that guide to their knowledge alone no more than that.
>
> Therefore**, those methods are in the eyes of the students as a dream with no interpretation**, since they do not know the way, the issue is realized through that method, and how was it the reason for its existence and **they grope for the wall like the blind** [Isaiah 59, 10].
>
> - The second reason is that when the reasons for the methods that guide to their knowledge are hidden from the students, they know only the methods of the author whose methods they are studying.
>
> However, they do not know the source and origin of the methods of all other authors, indeed those methods are sealed and concealed, none comes out, and none comes in [Joshua 6, 1].
>
> And even more, he will not be able to innovate on his own a method and trick in the questions of this science, he will not be able to answer questions other than those that he dealt with by himself."

Also, later (not only in the introduction) Mizrahi refers to didactic issues. Here are some examples:

i. **What is the desired teaching order?** From simple topics to complex ones or vice versa?

We find an extensive reference to the subject in Comtino's book (Comtino was Mizrahi's teacher): although he chooses an approach from the simple to the complex, he quotes Aristotle who also spoke of the advantages of the opposite method (Comtino, f. 26v).

> We should inform why we precede [the discussion on] the addition of a number to a number before the rest of the operations.
> **I say that it is because it is simpler than them and it precedes them by nature**, since it is simpler than them, as each of them consists of it and of the property that is specific for it; …
> One cannot say that because it is simple, it should follow the composite, according to Aristotle's explanation at the beginning of Physics that the general should precede the special, for three reasons…

Mizrahi does not hesitate at all, and it is clear to him that the "simple" must be taught first before the "complex" (Segev, p. II-15):

> The first chapter will deal with adding numbers. And indeed, we started by adding numbers among the other operations because it is the simplest one.
> …
> Since the simple ones come before the complex ones, we will be required to take this operation before the other operations in numbers.

ii. How to choose examples?

When there are different algorithms for the same operation, Mizrahi chooses the same example (the same numbers) to facilitate the comparison between the methods, for example in the first four multiplication methods he presents (Segev, p. II-34).

> The example of multiplying two-digit numbers by two-digit numbers is the example mentioned above itself so that you can differentiate between one way and another, and that's it.

iii. Which algorithm is preferable?

In most arithmetic operations, Mizrahi brings a large variety of algorithms. He is not satisfied with just bringing, them but compares them and analyzes their advantages and disadvantages.

In integer multiplication, for example, after presenting the first method (the same method used today), he points out that this method requires memorization and therefore he brings another method that does not require memorization. There is still the possibility of making mistakes when using these methods if you do not take care to place the numbers correctly. Therefore, another method, Gelosia, is presented later, where the chance of mistakes is smaller.

Summary and conclusions

In the 12th-16th centuries, mathematics studies in Europe took place mostly in universities (usually with a theoretical approach - Euclid and Boethius) and in reckoning schools (practical studies intended for merchants and artisans) (Bjarnadóttir, 2014; Høyrup, 2014).

In contrast, the Jews usually studied within their community. The sources of the Hebrew texts, according to the testimonies of the writers themselves, are texts in Arabic mainly, sometimes Latin or Greek that were found in Spain, Italy, or Byzantium. They sometimes read these texts in their original language and sometimes in Hebrew translation. Almost no evidence was found that they knew books written at the time in other European languages (French, German, etc)[8].

There is evidence of a small number of translations of Jewish texts into Latin such as a text of Bar Hiyya - Hibbur ha-Meshihah ve-ha-Tishboret, translated in 1145 into Latin by Plato of Tivoli as Liber embadorum, or the partial translation of Mizrahi's "Book of the Number" by Schreckenfuchs in 1546.

Most of the authors of the Hebrew mathematics books were experts not only in mathematics but also in other subjects: religion, philosophy, other sciences, and in addition didactic issues. Their views on the didactic issues present a broad and interesting picture, relevant even today.

Most of the books are written with a practical approach that focuses on acquiring the skills to perform various calculation operations or to calculate areas of geometrical shapes. These practical books are generally written in a concise manner and without proofs or lengthy explanations. Some also point to the moral purpose of preventing injustice due to ignorance of arithmetic. Exceptions are Levi ben Gershom's and Eliyahu Mizrahi's books, which emphasize the need to study proofs not only for mathematical accuracy but also to improve and deepen the learner's knowledge and mathematical skills.

Another interesting group of writers is those who have an intellectual interest in mathematics and are interested in studying it mainly for themselves.

In this article an attempt was made to present the didactic concepts of Hebrew mathematicians in 12-16th century. Additional issues related to the subject require further research: comparisons with books on the same subjects written at the same time, the development of Hebrew mathematical literature after the 16th century, and more.

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[8] Aradi (2015) mentions two manuscripts that may be linked to German texts on arithmetic.

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