

# Teaching Engineers in Spain in the XVIII century: Analysing Mathematical Courses

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## Abstract

*The analysis of the treatment of new mathematics in the training of engineers and gunners, taught in private colleges, academies and universities in the XVIII century, merits attention for a better understanding of the evolution of the history of mathematical education in this environment. We consider herein the activity of leading teachers and scholars who, despite being side lined in the common historiography of mathematics, played a major role in disseminating “modern” mathematical theories.*

*In Spain, as in other countries, the origins of “scientific” engineering settle in the Army, where officers acquired their training informally. After the War of Succession (1701-1714), the Spanish Bourbon Monarchy played a leading role in scientific and technologic development by establishing institutions to provide higher education for the officer corps, and in these academies mathematics constituted a core subject.*

*In 1739, a Royal Ordinance established the contents of the course in mathematics to be taught in the academies. This course, prepared by Pedro Lucece (1692-1779), consisted of eight treatises with a total of approximately 2,200 pages devoted to the main fields of mathematics, including “pure” mathematics (arithmetic and geometry), and “mixed” mathematics (cosmography, statics, hydraulics, architecture, artillery, and fortification). The aim of this paper is therefore to analyse the treatment given to pure mathematics in this original course and to highlight some ideas which it contains. In addition, we make some reflections and provide evidence on the process of transition from workshop technical culture to school culture. We briefly present a notable case of this transition, the private mathematical course prepared by the Minorca artist Pasqual Calbó.*

Keywords: history of education, Mathematical Courses, XVIII century Spain<sup>1</sup>

## Mathematics and the training of engineers and architects

Until the XIX century, no clear division existed between mathematics and the subjects using mathematics. The two categories were pure and mixed mathematics: pure mathematics included Arithmetic, Geometry, Algebra (from the mid-XVII century), and Calculus (from the mid-XVIII century); as well as Logarithms, plane and spherical Trigonometry. Mixed mathematics, also called Physics-mathematics, included a great variety of disciplines (Artillery, Optics, Perspective, Astronomy, Geodesy, Mechanics...), and even Experimental Physics (at the end of the XVIII century). Dear (2011) analyses the process of definition of mixed mathematics, a term extracted from Aristotelian conceptions that was not used by some authors, such as the Jesuit Claude François Milliet Dechaies (1621-1678), who preferred

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“physico-mathematics”. During the first half of the XIX century some new disciplines emerged from mixed mathematics: such as mathematical physics, electricity and magnetism, thermodynamics, and also what was known as “applied mathematics”.<sup>2</sup>

This grouping of disciplines around mathematics was usually linked to the training of technicians, mainly military engineers or architects.

### **From the apprentice – master system to the regular schools**

In practical professions - builders, artisans, miners, and some ship-owners and seafarers - the craft was learned through personal contact between apprentice and master. In fact, this system is still alive in certain fields. In Army, aspirants to technical positions learned their skills in this way of personal contact until the establishment of training schools, as is the case of the artillery school in Seville in the early XVI century (Cossart, 2021), which was an exception at this time.

A slow process throughout the XVI-XVIII centuries led to the founding of regular schools with a curriculum, a team of teachers and a timetable. In fact, it is interesting to note that historiographical debate is at present being held on the origin and process of the professionalization of engineering.<sup>3</sup>

The school of engineering at Leiden University<sup>4</sup> that Simon Stevin (1548-1620) was asked to set up in 1600 is often mentioned in this regard. Among other contributions, Stevin promoted the use of decimal numbers as the most useful for practical purposes.

Nevertheless, the process from workshop culture to school culture was not linear, giving rise to different experiences and engendering attempts to find a different orientation.

We propose herein to consider some intermediate stages in order to understand this transition from master-apprentice training to the foundation of formal schools. At a certain point in time, some private courses by teachers unattached to any institution were organised, with the aim of complementing the personal training that followed a handbook (books often issued from courses). At an early stage, these private courses had no formal connections with academic institutions, but were subsequently given inside institutions, although without either entrance qualifications or the awarding of diplomas.

Some known examples of these two intermediate situations are for instance, that of Jacques Ozanam who gave private mathematical courses that were reflected in handbooks, while some teachers of mathematics at French colleges organised parallel courses that went further than the limited official curricula. This was the case of the preparatory courses for joining the Army, for example, in France, entrance exams were held for admittance to some Corps. Later on, some of these

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<sup>2</sup> See Brown, 1991. For the English case, see Warwick, 2003.

<sup>3</sup> Kranzberg (ed.), 1986 contains several approaches to the setting up of technical schools in different countries.

<sup>4</sup> Minnaert, 1981.

teachers organised the same courses inside the colleges, sometimes referred to as *Chaires royales de mathématiques et d'hydrographie* (Royal Chairs of Mathematics and Hydrography) (De Dainville, 1954).

Therefore, four distinct stages may be considered:<sup>5</sup>

- **A) Apprentice – master training**
- **B) Private courses**
- **C) Private courses welcomed in educational institutions**
- **D) Formal technical schools**

In all these stages, but mainly in stage B, as mentioned above, some courses were published in handbooks, known at the time as “mathematical courses” and based on the manuscripts of those courses.

## Mathematical courses

The mathematical courses constituted scientific and encyclopaedic works that appeared during the 17th, 18th and 19th centuries and dealt with pure and mixed or physics-mathematics. Christian Wolff (1679-1754) included a survey of the mathematical courses printed before his own course.<sup>6</sup> Wolff states that one of the earliest published mathematical courses was Pierre Hérigone’s course.<sup>7</sup> These mathematical courses consisted of multivolume textbooks designed for to teaching and whose purpose was to spread the knowledge of the mathematical disciplines as a practical, scientific and technical activity.

The main mathematical courses in the seventeenth century were those in France by Pierre Hérigone (1634/1637/1642), Milliet Dechales (1674-1690) and Jacques Ozanam (1693), while those in the eighteenth century were in Spain, by Tomás Vicente Tosca (1707-1715), in Germany, Christian Wolff (1713-1715) and in France, Bélidor (1725).<sup>8</sup>

## Spain after the War of Spanish Succession

The War of Spanish Succession (1700-1714) provided the opportunity to formalize the Corps of Military Engineers (1711) as a technical organization. The Corps was proposed by the engineer Jorge Próspero de Verboom (1667-1744)<sup>9</sup> who had received mathematical training in a military Academy set up in Brussels by

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<sup>5</sup> A general perspective in Gouzévitch et al., 2004. Roca-Rosell, 2019, includes the Spanish case. See also: Lusa & Roca-Rosell, 1999.

<sup>6</sup> Wolff, Christian (1713-1715): *Elementa Matheseos Universae*, Hale, Magdeburg, Officina Libraria Rengeriana, V volumes. See: “De Praecipuis scriptis mathematicis brevis commentatio”, V, 3-21.

<sup>7</sup> On Hérigone’s course, see: Massa-Esteve, 2008 and Mellado, 2022. Hérigone’s date of birth is not known, probably 1580. He died in 1643.

<sup>8</sup> On Tosca, see: Navarro Brotons, 1985, 2003.

<sup>9</sup> Jorge Próspero de Verboom was born in Brussels in 1667. He was educated in the Academy of Brussels directed by Sebastián Fernández Medrano. In 1692, he succeeded his father as an important engineer in the Low Countries. It seems that he met the French engineer Sebastien de Vauban. In 1709, he was in charge of the creation of the Spanish Corps of Military Engineers, approved in 1711. He died in 1744.

Sebastián Fernández de Medrano (1646-1705). This Academy owed its existence to the personal initiative of this officer, who was convinced that a military officer should have undergone a comprehensive military training. As we will explain later, Verboom followed in the footsteps of his master and proposed the creation of a series of military academies devoted to mathematics.

The Bourbons, the new royal dynasty in Spain, promoted the renewal of training for naval officers, and in 1715 an Academy for *Guardamarinas* (marine officers) was set up in Cádiz. Like its antecedent in France (Rocheport), this school was created in order to prepare soldiers for a fighting unit engaged in missions of war, although training activities at this Academy were low due to these self-same engagements. However, after 1752, and under the supervision of Jorge Juan (1713-1773), the Academy became a centre of higher technical education, and the Crown was committed to an overhaul of the Army.

During the XVIII century, several attempts were made in Spain, outside the army, to train artisans and other technical men through the introduction of mathematical courses (Garma Pons, 2002).

The aim of our research group is to analyse some particular cases: the first of which deals with two military courses: The Mathematical Course at the Military Academy of Barcelona, and the Military Course of Pedro Padilla (presented in this volume by Mònica Blanco). Secondly, the following two non-military experiences: The Public Chair of Mathematics in Barcelona and the private course by the Minorca painter Pasqual Calbó.

### **The Public Chair of Mathematics of Barcelona (1756)**

In Barcelona, the Society of Jesus promoted the creation of a public chair of mathematics, i.e., a chair providing teaching to all social classes, mainly artisans. The Jesuit Tomàs Cerdà (1715-1791) went to Marseilles to learn from the experience of Esprit Pezenas (1692-1776), who was in charge of the Observatory in the city as well as occupying a Military Chair of Hydraulics. As director of a centre corresponding to our stage B (Private courses), Pezenas is a noteworthy example of the steps taken towards the creation of formal technical schools. In 1756, back in Barcelona, Cerdà found himself in charge of the new training provided at the Public Chair of Mathematics, created under the auspices of the municipality of Barcelona with the backing of the Crown. Cerdà drew up a mathematical course on both pure and mixed mathematics. This Public Chair was linked to the Noble College of Cordelles, where Cerdà was teacher (Cerdà, 1758, 1760, 1764), which furnishes us with a further case of stage C, a private course taught in another institution.

Cerdà published three treatises of his course: Arithmetic and Algebra in two volumes; Geometry in one volume; and, also, a treatise on Artillery, in this case in connection with the creation of a Military School of Artillery in Segovia (Navarro Loidi, 2008 and 2013). Given that Cerdà left Spain in 1767, the year in which the Society of Jesus was expelled from the country, his manuscripts were passed into the hands of the Royal Academy of History in Madrid. Two of these treatises have

recently been published, one on Astronomy and the other on Fluxions (Gassiot, 1998; Berenguer, 2015, 2016 and 2021).

### **Pasqual Calbó and his mathematical course (1803)**

An extensive manuscript by the painter Pasqual Calbó i Caldés (1752-1817) can be found at the Museum of Menorca in Maó. Calbó was a remarkable artist and the fact that he was at the same time the author of a text on mathematics, architecture and experimental physics made him more intriguing. The text was probably completed in 1803.

We have recently deduced that this manuscript is in fact the notebook of a mathematical course taught by Calbó. It is important to note that Calbó spent eight years in Venice and Rome thanks to a grant from the Austrian Empire. He was subsequently employed as a “Delineatori” (expert on drawing) at the new imperial art gallery in the Belvedere Palace, although he later left this position and returned to Minorca.

His manuscript consists of pure and mixed mathematics: The pure mathematics included decimal fractions, proportions, geometry, algebra, logarithms, and plane and spherical trigonometry, while the mixed mathematics included experimental physics (mechanics, cosmography, electricity, gases, optics, sound), sundials, perspective, civil and military architecture, and boat building. The text states that the course was aimed at the “young craftsmen [*menestrals*] of Minorca”, a clear indication of its emphasis on training. The text also refers to its practicability, far from theoretical purposes.<sup>10</sup>

While some contextual references help us to determine Calbó’s sources, for the sections on pure mathematical, Vicente Meavilla (2020) finds that parts of the courses by Tomás Vicente Tosca (1651-1723) and Benet Bails (1730-1797)<sup>11</sup> were used, some of which are translated, but somewhat abridged. The section devoted to the experimental physics contains an explicit reference to the course by Giuseppe Saverio Poli (1746-1825).<sup>12</sup> It is very interesting to note that Calbó decided to include Experimental Physics as part of a mathematical course, a relevant phase in the transition from a purely “entertaining” activity (for example, the experiments in electricity) to a mathematical theory in the XIX century (Heilbron, 2011; Dear, 2011).

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<sup>10</sup> Sintes y de Olivar, 1987, p. 87-101 published the correspondence between the Austrian Chancellor Wenzel Anton von Kaunitz-Rietberg, the tutors of Calbó in Venice and Rome, Jacoppo Durazzo and Giovanni Francesco Brunati, and Calbó in the period 1774-1779.

<sup>11</sup> It should be remembered that Minorca was under British administration (1713-1756), French administration (1756-1763), British administration (1763-1782), Spanish administration (1782-1798), British occupation (1798-1802), and finally, it was incorporated to the Spanish kingdom (1802).

<sup>12</sup> Calbó i Caldés, 2020, 2023.

<sup>13</sup> On Bails’ course, see Martínez Verdú, 2017, Martínez-Verdú & Massa-Esteve, 2021 and Martínez-Verdú et al., 2023.

<sup>14</sup> On Calbó and experimental physics, see: Jordi et al., 2023.

In the treatises on perspective, there are a few references to architecture and boat building, but the research work at present in progress points out that Calbó drew on some highly significant sources.<sup>13</sup>

Despite the fact that some details point to the possibility that the manuscript was meant for publication, Calbó's course remained in a manuscript form. The local government of Minorca officially declared 2017 as "Calbó's Year", for which an anthological exhibition was organized and approval given for a critical edition of the text. One should also add that Calbó's manuscript is written in Minorcan, the dialect spoken on the island, a significant event for Catalan culture. After the War of Spanish Succession, in which the Castilian army defeated the army of the Kingdom of Aragon (Catalonia, Aragon, Valencia and the Balearic Islands), the Catalan institutions were dismantled and the use of the Catalan language was banned. Minorca was an exception, since the Island became a British possession after the Treaty of Utrecht, and remained under British administration throughout most of the XVIII century, during which time its existing institutions were upheld and no restrictions were imposed on the use of the Catalan language. This situation changed after 1802 when Minorca once again came under Spanish administration, and consequently Calbó's text constitutes a unique example of technical training in Catalan.

This course provides us with an example of stage B, a private course for the training of technicians. During this time Minorca was undergoing a significant phase of development as a result of naval activity and the expansion of new urban building. There was also a fresh demand for technicians to work on the harbour and architects for the design of new buildings.

### **Military training: The Barcelona Royal Military Academy of Mathematics (1720)**

In 1720, a Military Academy of Mathematics was set up in Barcelona, promoted by the engineer-in-chief of the Corps, Verboom, who proposed that the Crown should create a network of academies with this objective in mind. A document by Verboom on the administration of the academies (Verboom sought to create an academy in each province), dated 1730, provides information about his ideas on military education. According to Verboom, students required a theoretical and practical education in:

Universal Geography, celestial and terrestrial; in particular, terrestrial and maritime; the three Architectures: Military, Civil and Hydraulics, the moving forces and all kinds of Machinery, compressive and elastic, Optics and Perspective, Conic sections from which all the above-mentioned architectonic subjects depended. The Science of Projections, planes and profiles, all of which made possible all types of buildings, constructions and

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<sup>14</sup> On Calbó and experimental physics, see: Jordi et al., 2023.

<sup>13</sup> This would be published in the third volume of the edition of Calbó's manuscript. The studies were undertaken by Joaquim Guerola, Margarita Galceran, Judit Garriga, Joan-Lluís Torres and Fátima Romero.

machinery. The principles of Astronomy and Art of Navigation for the effective execution of all kinds of marine buildings, and finally the Theory and Experimental Physics of natural things, without which knowledge the universal and excellent profession of Engineer would not only be limited but also discredited...<sup>14</sup>

Mateo Calabro<sup>15</sup> was initially appointed as director of the Military Academy of Barcelona. He was responsible for the preparation of a course on mathematics, the content of which is described in his letter of 1724 to the Count of Montemar. In 1737, after a controversy between the director and Verboom, a new director was named, Pedro Lucuce (1692-1779). He prepared a course (unpublished) that was taught for more than 40 years and has been preserved thanks to the copies made by several pupils<sup>16</sup>.

Lucuce's *Mathematical Course for Military Instruction*<sup>17</sup> (1739-1744) consists of eight treatises comprising of 2,200 pages and dealing with the following subjects: Treatise I: On Arithmetic. Integers, linear algorithm; ratio and proportion usual [*in common*]; rules of proportion; powers and roots, and progressions (270 pages). Treatise II: On Elementary Geometry. Euclid's *Elements*; an appendix on conic sections (331 pages). Treatise III: On Practical Geometry. Plane trigonometry; the logarithms; the construction of plane figures; the inscription and circumscription of straight figures in the circle; transformation of plane figures; the use of instruments; planimetry; stereometry; levelling (296 pages). Treatise IV: On Fortifications. Regular fortification; irregular fortification; fortifications effective in the field; campaign fortifications (315 pages). Treatise V: On Artillery. The nature, composition, inspection and conservation of gunpowder; ancient and modern artillery, drawing of mortars, canons, gun carriages and weapons; canon and mortar batteries; fireworks, artillery trains and inventories of strongholds (372 pages). Treatise VI: On Cosmography. Celestial spheres, geography, hydrography, seamanship and time measuring (392 pages). Treatise VII: On Statics; the motion of heavy bodies; machinery; hydraulics; optical compendium; general principles of optics; perspective (300 pages). Treatise VIII: On Civil Architecture. Building decoration and beauty; the solidity and safety of constructions (270 pages).

At the beginning of the course, Lucuce explained the mathematics he intended to use, and divided it into pure and not pure or mixed [*impuras*], the last also called physic-mathematics:

Mathematics is divided into various parts, which are reduced to two types, one pure and the other mixed; pure mathematics is Arithmetic and

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<sup>14</sup> Verboom, "Proyecto o Idea", 1730, p.18 and the transcription in Roca-Rosell & Massa-Esteve, 2020.

<sup>15</sup> The Barcelona Academy of Mathematics had seven directors: Mateo Calabro (1720-1738); Pedro de Lucuce y Ponce (1738-1779); Claudio Martel (1756-1760); Juan Caballero y Arigorri (1779-1784); Miguel Sánchez Taramas (1784-1789); Félix Arriete (1790-1793) and Domingo Belestá y Pared (1794-1802). See Capel et al., 1988.

<sup>16</sup> For a global perspective on history of mathematics teaching, see Schubring & Karp (2014).

<sup>17</sup> *Curso Mathematico para la Instrucción de los Militares*.

Geometry, because they deal with quantities regarding what is numerable or measurable; the others are physico-mathematics, because they consider quantities accompanied by some accident or perceptible attribute [*sensitive affection*] typical of physics, such as optics, which deals with visible quantities and the music of sound<sup>18</sup>.

The name introduced by Lucuce for referring to mixed mathematics, like Tosca in his *Compendio*, is physico-mathematics. In the introduction to his Mathematical Course, Lucuce remarks on the relevance of Aristotle's physics, which illuminates many other natural sciences by means of mathematics, in accordance with the introduction to Tosca's *Compendio*. He stated as follows:

The desire to know (said Aristotle) is natural to men, and among all the natural sciences that which gives most satisfaction is Mathematics, for the lucidity of its truths, the energy of its proofs, and the clarity of its demonstrations. According to its Greek derivation, Mathematics is the same as Doctrine, or discipline, since it is exempt from the doubts and opinions so frequently found in the other sciences, for which reason whosoever devotes time to its study will not be disappointed, nor will the sweat shed in such an agreeable pursuit be in vain: therein the studious will find delight; since those fogs that often obscure the brilliance of other faculties do not cloud the sublime reaches of mathematics: rather such lights shine down from its soaring spheres to illuminate the pathways of the other natural sciences and lead to unmistakable truths<sup>19</sup>.

In the same introduction to his Mathematical Course, Lucuce also emphasized the usefulness of mathematics in physics, optics, astronomy, the construction of buildings and the organization of armies, among other subjects. In his own words:

With her (mathematics) the most closely guarded secrets of nature are revealed; she is the one that investigates the forces of impetus, the conditions of movement, the causes, effects and differences of sounds, the nature of light and its means of propagation. She raises buildings in beauty, makes cities well-nigh impregnable, orders armies with admiration and opens horizons to seafarers; lately she has ascended to the heavens to chart the grandeur of the Stars and the harmony of their movements...<sup>20</sup>.

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<sup>18</sup> "Dividiré la mathematica en varias partes, que se reducen a dos especies: la una de puras y la otra de impuras: las puramente Mathematicas son la Arithmetica, y Geometria, porque tratan de la cantidad enquanto numerable o mensurable; las demás son Physico Mathematicas porque consideran la cantidad acompañada de algún accidente o afección sensible propio de la Phisica, como la "obtica" que trata de la cantidad visible, y la Música de la sonora". (Lucuce, introducción, s/n).

<sup>19</sup> "Es natural en los hombres (dixo Aristoteles) el deseo de saber, y entre todas las ciencias naturales la que más le satisface es la Mathematica, por la limpieza de sus verdades, energía de sus pruebas, y claridad de sus demostraciones. Mathematica según la deribazion del Griego es lo mismo que Doctrina, o disciplina, por carecer de las dudas, y opiniones tan frequentes en las demás ciencias, por cuja razón no será malogrado el tiempo que se dedicare a su estudio, ni el sudor que se empleare en tan ameno campo: en el que experimentará el estudioso sus delicias; pues no llevan a la excelsa región de la mathematica aquellas nieblas que suelen oscurecer el resplandor de otras facultades: antes bien descenden de su remontada esfera tales luzes, que manifiestan las sendas a las otras Artes naturales para hallar la verdad con acierto". (Lucuce 1739-1744, Introducción, s/n).

<sup>20</sup> "Con ella se descubren los más ocultos secretos de la naturaleza, ella es la que aberigua la fuerza del impetus, las condiciones del movimiento, las causas, efectos, y diferencias de sonos, la naturaleza de

## Analyzing Treatises of Pure Mathematics in Lucece's Course

de que se compone cada Potencia, quando la xavi está dividida en 2 partes como ay b.

**Tabla Synthetico Analítica**

a	b									
a <sup>2</sup>	2ab	b <sup>2</sup>								
a <sup>3</sup>	3a <sup>2</sup> b	3ab <sup>2</sup>	b <sup>3</sup>							
a <sup>4</sup>	4a <sup>3</sup> b	6a <sup>2</sup> b <sup>2</sup>	4ab <sup>3</sup>	b <sup>4</sup>						
a <sup>5</sup>	5a <sup>4</sup> b	10a <sup>3</sup> b <sup>2</sup>	10a <sup>2</sup> b <sup>3</sup>	5ab <sup>4</sup>	b <sup>5</sup>					
a <sup>6</sup>	6a <sup>5</sup> b	15a <sup>4</sup> b <sup>2</sup>	20a <sup>3</sup> b <sup>3</sup>	15a <sup>2</sup> b <sup>4</sup>	6ab <sup>5</sup>	b <sup>6</sup>				
a <sup>7</sup>	7a <sup>6</sup> b	21a <sup>5</sup> b <sup>2</sup>	35a <sup>4</sup> b <sup>3</sup>	35a <sup>3</sup> b <sup>4</sup>	21a <sup>2</sup> b <sup>5</sup>	7ab <sup>6</sup>	b <sup>7</sup>			
a <sup>8</sup>	8a <sup>7</sup> b	28a <sup>6</sup> b <sup>2</sup>	56a <sup>5</sup> b <sup>3</sup>	70a <sup>4</sup> b <sup>4</sup>	56a <sup>3</sup> b <sup>5</sup>	28a <sup>2</sup> b <sup>6</sup>	8ab <sup>7</sup>	b <sup>8</sup>		
a <sup>9</sup>	9a <sup>8</sup> b	36a <sup>7</sup> b <sup>2</sup>	84a <sup>6</sup> b <sup>3</sup>	126a <sup>5</sup> b <sup>4</sup>	126a <sup>4</sup> b <sup>5</sup>	84a <sup>3</sup> b <sup>6</sup>	36a <sup>2</sup> b <sup>7</sup>	9ab <sup>8</sup>	b <sup>9</sup>	
a <sup>10</sup>	10a <sup>9</sup> b	45a <sup>8</sup> b <sup>2</sup>	120a <sup>7</sup> b <sup>3</sup>	210a <sup>6</sup> b <sup>4</sup>	252a <sup>5</sup> b <sup>5</sup>	210a <sup>4</sup> b <sup>6</sup>	120a <sup>3</sup> b <sup>7</sup>	45a <sup>2</sup> b <sup>8</sup>	10ab <sup>9</sup>	b <sup>10</sup>

**Coscilio 1<sup>o</sup>**  
Supuesto que la 1<sup>a</sup> potencia sea a y la 2<sup>a</sup> b se hallará que la Potencia 10<sup>a</sup>

Figure 1. Synthetic Analytic Table with the powers of  $(a + b)$  until the tenth. Lucece's Course (1739-1744), 105v.

Lucece began Treatise I with the definition of Arithmetic as a science that dealt with discrete quantity; that is, the numbers, and is divided into Speculative and Practical. He made a further classification by reason of his characters divided into Vulgar and Literal, stating that:

By reason of the notes, numbers or characters that Arithmetic uses, it is divided into Vulgar and Literal; the first exercises its operations by the vulgar notes, and the second by the literals or letters of the Alphabet; also called "specious" [referring to Viète's speciosa] because its doctrine can be applied to geometry and other mathematical sciences. Both will be addressed by dividing it into 6 books<sup>21</sup>.

la luz, y el modo de su propagación; levanta con hermosura los edificios, y hace casi inexpugnables las ciudades, ordena con admiración los exercitos, y abre camino a los navegantes; últimamente se remonta hasta el cielo para averiguar la grandeza de los Astros, y armonía de sus movimientos..."(Lucece 1739-1744, Introducción, s/n).

<sup>21</sup> "Por razón de las notas, cifras o caracteres de que se sirve la Arithmetica, se divide en Vulgar, y Literal; la primera exercita sus operaciones por las notas vulgares, y la segunda por las Literales o letras del Alphabeto; llamase también especiosa por que su doctrina se puede aplicar a la Geometría, y demás ciencias Mathematicas: de una y otra se tratará dividiéndola en 6 libros". (Lucece 1739-1744, 6).

Thus, the contents of Treatise I consist of six books: 1) Vulgar Arithmetic; 2) Literal Arithmetic; 3) On Ratio and proportion (in fact Book V of Euclid's *Elements*); 4) The rule of three; 5) The composition of the powers and the root extraction; 6) The progressions.

In this treatise, we find some novelties such as tables of powers with letters, quoting the notation from the Arab mathematicians, Diophantus and Viète. Moreover, we also find a table referred to as a Synthetic Analytic Table, which reproduces the powers of binomials until the tenth power with letters<sup>22</sup>(see Fig. 1).

### Euclid's *Elements* in Lucuce's course

In 1739, the reference in the Ordinance of the Royal Academy is that Euclid's *Elements* should form part of the syllabus (*Ordenanza*, 1739, 8). Consequently, in his introduction to Treatise II, entitled "On Elementary Geometry" and dealing with Euclid's *Elements* and Conic sections, Lucuce clarifies which books of the *Elements* he wishes to work with and in which part are explained:

Since the work is extensive and diffuse, in this treatise we explain Books 1, 2, 3, 6, 11 and 12; with respect to Book 4, it is addressed in Practical Geometry and Book 5 in Arithmetic, while the others, being of little use, are omitted<sup>23</sup>.

The decision to explain only Books 1 to 6 and 11 and 12 was usual at that time, because these books were considered to be the most useful. In fact, Tosca in his *Compendio* and Dechales in his *Cursus* explain eight books, specifying that Books 7 and 8 correspond to 11 and 12, respectively. Lucuce's originality resided in the fact that he preserved the Euclidian order, underscoring this idea as follows:

The order I follow in the propositions is the same as that given by Euclid, so that they may be cited whenever necessary, the most useful being demonstrated with all possible brevity and clarity in order to save time for the explanation of other subjects that are of concern for the instruction of military personnel<sup>24</sup>.

He went on to say that the books of the *Elements* provide proofs by lines, by letters and by numbers, thereby avoiding rhetorical discourse. Thus, in the introduction to Book II of the *Elements* in Treatise II of the course, he stressed that the comprehension of these constructions with letters are useful for understanding Algebra (see Fig. 2):

In this book we consider the rectangles and squares which are formed by dividing a rectilinear line into parts; its comprehension is of great usefulness in Mathematics and especially for Algebra; and, although its theorems are

<sup>22</sup> In Tosca (1707-1715) we find a similar table of powers of binomials with numbers, not with letters.

<sup>23</sup> "Como la obra sea tan difusa y dilatada, explicaremos en este tratado los libros 1, 2, 3, 6, 11 y 12, respecto de que el 4º se dará en la Geometría Práctica, el 5º se dio en la Arithmética; y los demás se omiten por ser de poca utilidad". (Lucuce 1739-1744, Introduction).

<sup>24</sup> "El orden que seguiré en las proposiciones será el mismo de Euclides, para que se puedan citar cuando convenga; demostrando las más útiles con la claridad y brevedad posible; a fin de no malograr el tiempo, que se necesita, para la explicación de otras materias propias también a la Instrucción de los Militares". (Lucuce 1739-1744, Introduction).

obscure, the demonstrations will be facilitated by lines, as well as by literal calculus and by numbers<sup>25</sup>.

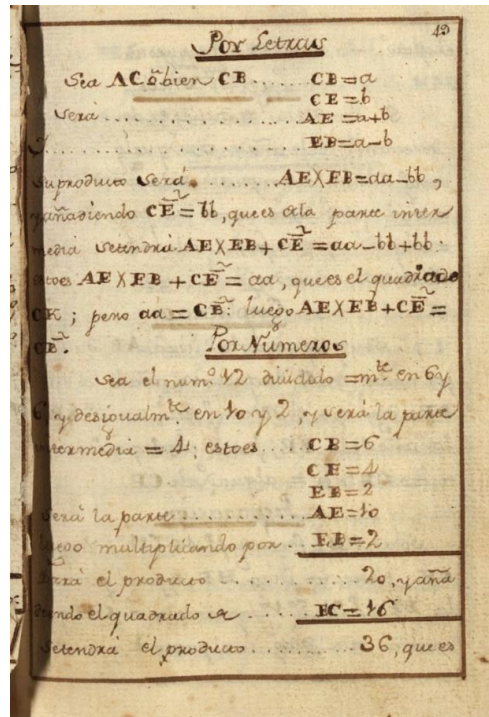


Figure 2. Lucuce's proof by letters and numbers. Lucuce's Course (1739-1744), 49r

We now consider the treatment of Book V on the theory of proportions, which is found in Treatise I. In an original and modern way, Lucuce moves the book on proportions from the second treatise dealing with geometry to the first treatise dealing with arithmetic. In Dechales' and Ozanam's courses, all the books in Euclid's *Elements* remain together, while Tosca's course only moves Book IV to Practical Geometry.

In a certain sense, Lucuce performs an arithmetization of the theory of proportions (Lamandé, 2013). Our aim is to characterize the arithmetization of theory of proportions that began in the XVI century. In the initial stage, some authors illustrated Euclid's definitions with numbers and/or algebraic symbols, while in the second stage proportions apply to all kinds of quantities, both discrete and continuous. The third stage can be represented by the identification of a ratio with a numerical value, and finally the proof of propositions on theory of

<sup>25</sup> "En este libro se consideran los rectángulos y quadrados que se forman sobre una línea recta dividida en partes; su inteligencia es de grande utilidad en la Matemática y con especialidad para el álgebra; y aunque sus teoremas son oscuros se facilitarán sus demostraciones assi por líneas como por el cálculo literal, explicándolas también por números". (Lucuce 1739-1744, 42).

proportions is performed by arithmetical lemmas. Lucuce worked with all the four stages of arithmetization. He illustrated with numbers and /or algebraic symbols. In addition, he applied proportions to all kinds of quantities, defined the ratio as the “exponent”, and finally used arithmetical lemmas in the proofs.

He argued that the theory of proportions can be applied to numerical ratios as well as to magnitudes, discrete and continuous, and that this is a necessary and universal key for comprehending mathematics, both pure and mixed, as a whole. So, at the beginning of Book III in Treatise I, he explained the relevance and the propaedeutic usefulness of the proportions as follows:

Book 3 (Treatise I). On ratio and proportion in common. This book, which is the 5th by Euclid, deals with the ratio and proportion in common, whose doctrine is suitable for all kinds of quantities, both discrete and continuous, which serve for numbers, lines, surfaces and solids; being the universal key to understanding the various parts that make up Mathematics<sup>26</sup>.

Once again Lucuce specified that the propositions on proportions explained in *Arithmetic* preserve the Euclidian order and are proved by letters and by numbers, thereby avoiding rhetorical explanations:

His propositions preserve Euclid’s, so they may be cited where appropriate; the (propositions) are less important and are omitted. Only the most helpful remain to be demonstrated by letters and explained by numbers in order to facilitate their understanding<sup>27</sup>.

Definition 5 of proportions presented by Lucuce is also original:

Definition 5. The exponent of the ratio is the quotient obtained by dividing the antecedent by the result. Example; if in the ratio of 6 to 2, 6 is divided by 2, the quotient 3 is the exponent which states the number of times that 6 contains at 2 and type 6/2.” (Lucuce 1739-1744, 59-60).

Lucuce uses this definition to show the relationship between ratios through the exponent of the ratio; that is to say, the numerical value of the ratio. "Corollary 2: The value of the ratio will be exposed by the exponent." Lucuce uses this definition to justify other definitions.

After the definitions and before the propositions, Lucuce presents two arithmetical lemmas to facilitate the demonstrations. The first lemma was also used by Viète to justify the solution of the construction of second degree. In Lucuce’s words:

Lemma 1. If four quantities be proportional, the product of the two extremes is equal to the product of the two means; and if the product of the extremes is equal to the means, the quantities are proportional. (Lucuce 1739-1744, 66).

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<sup>26</sup> “Libro 3º. De la razón y Proporción en común. En este libro, que es el 5º de Euclides se trata de la razón y proporción en común, cuia doctrina combiene a toda clase de cantidad ya sea discreta o ya continúa, esto es sirve para los números, líneas, superficies y sólidos; siendo llave universal para entrar en el conocimiento de quantas partes compone la Mathemática”. (Lucuce 1739-1744, 58).

<sup>27</sup> “Sus proposiciones guardan el orden de Euclides, para que puedan citarse cuando convenga; lasmenos principales se omiten y solo se darán las de mayor utilidad, que se demostrarán por letras, y se explicarán por números para facilitar su inteligencia”. (Lucuce 1739-1744, 58).

Therefore, in the demonstrations of the propositions Lucuce used the idea of the exponent or the arithmetical lemma quoted above. In light of the foregoing, we may say that Lucuce approached Euclid's *Elements* in an algebraic way.

### Concluding remarks

Military men, civil engineers, architects and craftsmen required mathematical resources in order to improve their skills and abilities. Cerdà's Course (that we consider in the stage C) had an ambitious orientation –for example, the inclusion of Calculus –and was followed by artisans, learned men and aristocrats, thereby paving the way to the creation of a Chair of Mathematics. Calbó's Course, on the other hand, provides us with a concrete example of a private activity in technical education (stage B). It would have been useful for training artisans for work in the harbour of Maó or in the building sector, both of which were very active at that time.

It is important to point out the emphasis placed on mathematics as the quintessential science for engineers at the Barcelona Royal Academy of Mathematics. But it should also be borne in mind that, although the Academy inculcated a view of mathematics that stressed the usefulness of the results, it still respected the classical authorities, such as maintaining the order and numbering of the definitions and propositions of Euclid's *Elements*.

Moreover, the version of Euclid's *Elements* presented in Lucuce's course is explicitly tailored to his audience (military engineers and artillerymen) in order to facilitate their comprehension and usefulness. While Lucuce confined himself to the explanation of only certain books belonging to Euclid's *Elements*, he nevertheless demonstrated Euclid's proofs by means of geometrical figures, by numbers and by letters, thereby avoiding rhetorical discourse.

Lucuce's course was taught for almost 40 years at the Royal Military Academy in Barcelona and had a great impact on the training of military engineers, as well as on the civil engineers who made use of this original course. However, the Barcelona Academy retained some characteristics of a traditional school; for example, there were rules for the admission of students, a curriculum established over 3 years, and a team of teachers, although no diplomas were awarded. Indeed, it could constitute an example of the path leading from C to D. We have also identified manuscript copies belonging to artisans, the existence of which suggest that Lucuce's Course was also followed in private classes.

In our paper, we have focused the mathematical teaching imparted at a “transitional” institution, the Barcelona Royal Military Academy of Mathematics, a centre of mathematical education for Army personnel.

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