

History of school geometry in the early years: Brazil, 19th and 20th century

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Abstract

This study aims to identify the interactions established between Brazil and foreign countries in the construction process of school geometry in the early years in the 19th century and at the beginning of the 20th century. To do so, we examined several sources: pedagogical programs, textbooks and pedagogical journals that circulated in Brazil in that period, as well as reports produced by Brazilian teachers who visited European countries on pedagogical missions at the end of the 19th century. The investigation was based on the theoretical postulate of the circulation of pedagogical knowledge, which was promoted by cultural agents, that is, educators, teachers, travelers, writers, exhibitors, etc. We aimed to analyze the appropriation processes regarding the knowledge received in Brazil by identifying its differences in comparison with the knowledge produced abroad (Burke, 2016).

Three examples of Brazilian appropriation of foreign content were analyzed in the incorporation of foreign proposals related to geometry teaching for the first years of school in Brazil. The three categories – drawing, handwork and tachymetry – selected for the appropriation analysis proved to be fundamental to understand a primary geometry teaching proposal. It is safe to say that school geometry in Brazil was fostered, collated and developed in an intimate connection with other disciplines or knowledge, or teaching methods, a result of close dialogue with several authors in a constant process of exchange, interchange, readings and interpretations of national and international experience, which reiterates the importance of pedagogical knowledge circulation (Fontaine, 2022).

Keywords: knowledge circulation, intuitive method, practical geometry.

Introduction

In Brazil, studies in the field of History of Mathematics Education have been considerably increasing in terms of production in undergraduate research, master's research, doctoral and post-doctoral research. The same holds true regarding articles published in Brazilian scientific journals and in researchers' meetings in national events, like *Encontro Nacional de Pesquisa em História da Educação Matemática* (ENAPHEM - *History of Mathematics Education Research National Meeting, in free translation*)¹, which have been taking place since 2012. Nevertheless, the participation of Brazilian researchers in the *International Conference on the History of Mathematics Education* is almost negligible.

Another characteristic of research performed in Brazil is related to collective projects around a specific theme, which gather a group of researchers from

¹ For further details, visit <https://enaphem.wordpress.com/>

different federal states in order to better investigate the Brazilian context, that is to say, a country of continental extension and significant social, economic and cultural diversity. The study we present here is a part of a larger² project that investigated mathematics teaching in primary school from the 19th century, right after the Independence of Brazil in 1822, until mid-20th century.

The objective of this study was to identify the interactions established between Brazil and foreign countries in the construction process of school geometry. Several sources were analyzed: pedagogical programs, textbooks³ and pedagogical journals that circulated in Brazil in the aforementioned period, as well as reports produced by three Brazilian teachers who visited European countries on a pedagogical mission at the end of the 19th century. This study continues the results presented by Silva & Leme da Silva (2020) by highlighting educational reforms related to geometry teaching at the international level (Barbin & Menghini, 2014) and reiterating proposals for an intuitive and experimental type of geometry that circulated internationally.

The first textbooks that circulated in Brazil originated in Europe or in the USA and were adapted or translated into Portuguese. Further evidence of international circulation were the pedagogical missions of teachers who traveled to Portugal, Italy, Sweden, Switzerland, England, France, Belgium and Spain. The reports produced by them describe what they observed and experienced in the countries in terms of pedagogical programs, pedagogical practices, textbooks and other aspects.

According to Fontaine (2022), “since the end of the 18th century, the circulation of pedagogical knowledge was intense both in Europe and towards the Americas, and it grew even more with universal exhibitions, pedagogical missions and especially education publications” (p. 6, our translation). Circulation was promoted by cultural agents, here understood as educators, teachers, travelers, writers and exhibitors, among others. The purpose was to analyze the appropriation processes of the knowledge received in Brazil, thus identifying its differences in relation to the knowledge produced abroad (Burke, 2016).

In this sense, Brazil was one of the countries that welcomed, consumed and appropriated foreign ideas and proposals in order to devise its first public instructions and to organize a republican primary school in the 19th century. Three examples of the Brazilian appropriation of foreign production were analyzed focusing on the process of incorporation of foreign proposals for geometry teaching in the first years of primary school in Brazil. The analyzed sources prioritized the state of São Paulo and Rio de Janeiro, which was the Brazilian Federal District at that time, since there was not a unified regulation for primary school in Brazil.

² Project developed in the scope of GHEMAT/SP, a research group in the History of Mathematics Education, São Paulo state, registered with Conselho Nacional de Pesquisa (CNPq – National Research Council, in free translation) in Brazil.

³ Textbooks are used in this context as a guide with instructions for teachers and activities.

Freehand drawing and Geometric drawing

The subject of geometry for the first years of primary school was born in Brazil in 1827, soon after the Independence (1822), when the first public instruction law was approved by the Brazilian Congress. The resolution stated that teachers had to teach general notions of practical geometry. The term “practical geometry” emerged as a result of debate among political representatives until they reached an agreement, as contained in the following extract from a debate among congressmen:

I do not want a master to teach or point what a straight line is. I want them to take the compass and describe a triangle over a line. It is not costly; it is the easiest thing to do (Ferreira França *apud* Moacyr, 1936, p. 183). Geometry principles are of the utmost necessity, even if one is going to be a carpenter or a constructor. (Cunha Mattos *apud* Moacyr, 1936, p. 184).

To satisfy the proposal for practical geometry, two years later, in 1829, A. F. de P. and Iollanda Cavalcanti d’Albuquerque⁴ published the textbook titled *Princípios do Desenho Linear compreendendo os de Geometria Prática pelo método do ensino mutuo*, extracted from L. B. Francœur⁵. It was a translation and adaptation of the French textbook *Le Dessin Linéaire, d’après la méthode de l’enseignement mutuel*, from 1819. The inclusion of the term *practical geometry* in the Portuguese title, not present in the French original version, reinforced the objective of obeying the Brazilian public instruction law.

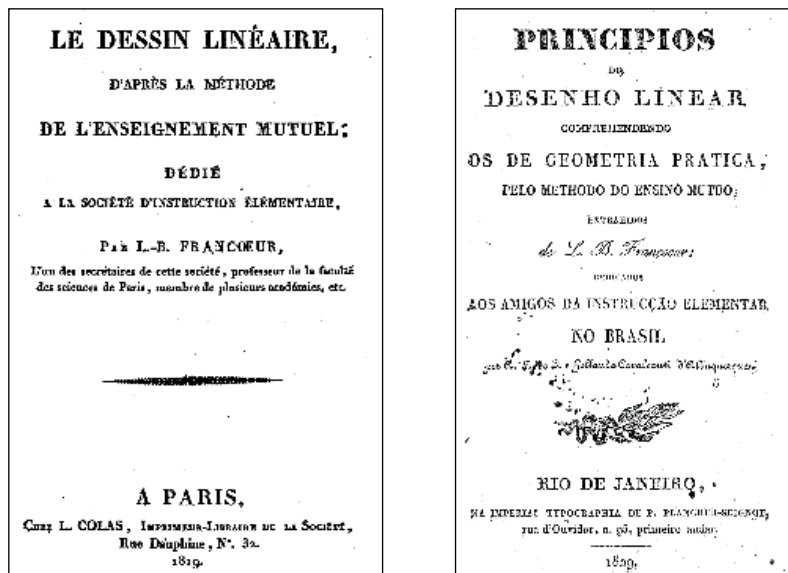


Figure 1 – Covers of Francœur (1819) and Albuquerque (1829)

⁴ A Brazilian politician.

⁵ A professor at Faculté des Sciences de Paris.

According to D'Enfert (2007), Francœur's textbook (1819) aimed to exercise the vision and the hand. Students should trace straight lines by hand before tracing curved lines, and then trace frames or ornaments. D'Enfert considers that the proposal was based on Pestalozzi's⁶ method, according to which knowledge is supported by a sensitive perception of nature and by visual observations.

The original textbook is structured in six parts and one section containing geometry and arithmetic problems. However, the Brazilian translation/adaptation only refers to the first four parts⁷. In the 1st part, tracing is always straight, starting with figures like straight lines, then angles and plane geometric figures. In the 2nd part, the traces form space geometric figures, but they are drawn with straight lines, according to figure 2.

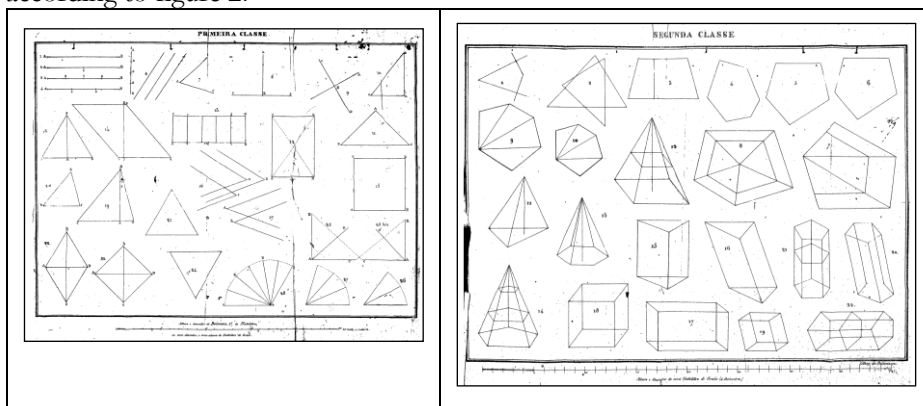


Figure 2 – Geometric figures from Albuquerque's textbook (1829) – parts 1 and 2

Circles and arcs are introduced in the 3rd part because students must have greater control of their hands in order to trace curved lines with plane geometric figures. In the 4th part, curved lines continue with space geometric figures. The sequence of the contents to be studied was not defined by geometric specificities but by the requirements imposed by freehand drawing of geometric figures, always aiming to reach great perfection.

Instructions for teachers contained in the French and Portuguese versions stated that teaching should happen without any precept established by the teacher: students should only make use of the "force of imitation". In other words, based on the practice of observation and reproduction, students should make consecutive freehand drawings until they precisely reached a perfect shape, which was verified by the teacher with measuring instruments.

⁶ Johann Heinrich Pestalozzi (1746-1826), a Swiss educator. Well-known by his work as a teacher, headmaster and school founder, his ideas defined intuitive pedagogy, whose basic characteristic was to provide students with sensitive objects for perception and observation (Zanatta, 2012).

⁷ The 5th part encompasses drawings that represent objects like a bowl and a flower vase, and the 6th part contains drawings that represent architectural ornaments.

Thus, the school geometry proposal for the first years of primary school in Brazil emerged in connection with the intuitive method and with freehand drawing of geometric figures, in which the practice of measuring and tracing characterized teaching. The name “practical geometry” is here understood as the exercise of students successively drawing geometric figures until perfection was reached.

In parallel, the Brazilian educational context in the 19th century encompassed a rudimentary primary school with lay teachers, inappropriate school space and groups formed by students of different ages and school grades in the same classroom. In 1893, after the Proclamation of the Republic (1889), the state of São Paulo inaugurated a new model of primary school that gathered separated schools, which was called “grupo escolar”. Seven teaching programs were proposed in the state of São Paulo until the mid-20th century, with different subjects. Those who contained geometry concepts were: Modeling (1894), Shapes (1894, 1925, 1934) and Geometry, Drawing and handwork, which were present in all programs (1894, 1905, 1918, 1921, 1925, 1934 and 1949/50).

The model symbolized teaching modernity and became a reference for other Brazilian states. With new physical and administrative organizations, a new presentation of teaching programs was devised, containing a detailed list of the subjects that should be taught and the standardization of school routines (Souza, 2009).

At the end of the 19th century, another textbook circulated in Brazil, but this time it was of American origin. It was *Primary Object Lessons* by Norman Allison Calkins⁸, in 1881. The textbook was not exclusively related to geometry; it contained proposals for lessons based on the intuitive method, encompassing different themes to be applied by parents and teachers. The textbook was adapted and translated into Portuguese by Ruy Barbosa⁹ and received the title *Primeiras Lições de Coisas*. It had great circulation, especially in São Paulo state.

Calkins’ translated textbook did not contain specific geometry lessons, but lessons on shapes and drawing. The lessons on shape focused on the observation and classification of objects, they were built through the senses and connected to a concrete experimental activity. They proposed simultaneous work with plane and space geometric figures at all times. Drawing lessons had the goal of developing pencil handling skills and the ability to identify figures and line positions, that is, freehand drawing practice, just as proposed by Francœur (1819) in his textbook.

⁸ Primary teacher and school headmaster in New York.

⁹ Rui Barbosa (1849-1923) was a lawyer, lecturer and journalist. His writings on education encompassed a limited period (1881-1886) of his political career as a social reformer (Bastos, 2000). Barbosa’s translation of Calkins’ book was published in 1886, but the edition analyzed in this paper was published in his *Obras Completas* (Complete Works, in free translation) in 1950.

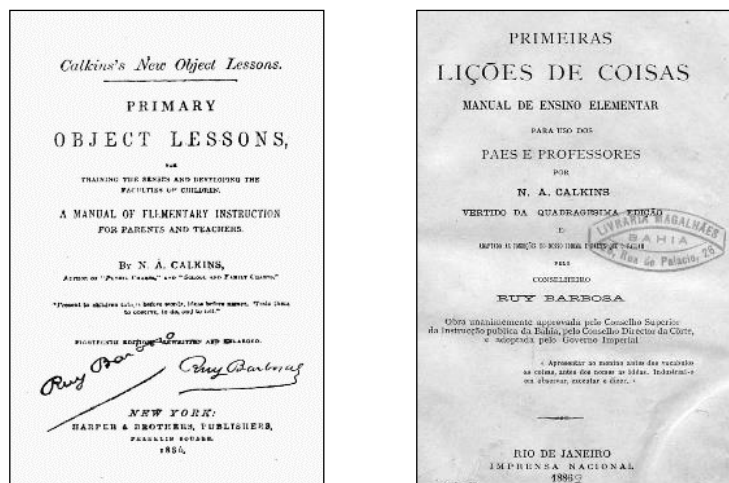


Figure 3 – Covers of Calkins – original (1881) and translation (1886)

Calkins' textbook did not make any reference to geometric drawing with instruments. However, Rui Barbosa, its translator, criticized the proposal concerning the insertion of geometric drawing into geometry teaching for children. According to Barbosa, "giving children ruler and compass right from the start means taking away all their will to learn and all their initiative; it means paralyzing their most precious sense – the sight" (Vasconcelos *apud* Calkins, 1950, p. 141, our translation).

At that time, the end of the 19th century, there seemed to be a consensus in Brazil about approaching geometric figures drawing practice for early geometry teaching. Nevertheless, two drawing proposals arose: freehand drawing through copy and imitation and geometric drawing with ruler and compass.

The textbook *Primeiras Noções de Geometria Prática* by Olavo Freire da Silva, one of the first Brazilian original productions, was published in 1894. Even though the expression "practical geometry" was used in the title, Freire da Silva's proposal (1907)¹⁰ did not mention freehand drawing; he provided readers with a true manual on geometric constructions presented as solved problems. It started with problem I – Building an angle like another given angle – and ends with problem XCII – Tracing a hyperbola with the compass following the given foci and vertices. They are constructions based on the use of ruler and compass with a step-by-step guide in the form of prescriptions as to make the drawings, as we can see in problem III, page 35, in the figure below:

¹⁰ We could not find the 1st edition from 1894. The edition here analyzed is from 1907, the 9th edition, with 490 exercises, 92 problems and 381 illustrations.



Figure 4 – Cover of Freire da Silva's textbook (1907) and page 35

The significant circulation of Freire da Silva's textbook was strengthened by the prescription of geometric constructions with ruler and compass in different pedagogical programs, such as the programs put in place in the states of São Paulo, Minas Gerais, Sergipe, Goiás and Rio Grande do Sul at the end of the 19th century and in the early 20th century (Leme da Silva, 2014).

Furthermore, at the beginning of the 20th century, *Revista de Ensino*¹¹, from São Paulo state, published that the debate over the proposals for drawing teaching was established in line with what was occurring abroad. The article *A Natureza, mestre do desenho. Bancarrota do ensino oficial*, (Nature, The Master of Drawing. The Fall of Official Teaching, in free translation) by Jorge Moreau, translated into Portuguese by Ruy Paula de Souza and published in 1906, revealed the discussions on the teaching of drawing at the international level and presented two different methods: sculptor Guillaume's geometric method and the natural method by philosopher Ravaisson:

At that time, the geometric method won, crossed the Atlantic and arrived in the USA, causing the same problems as in France: at the end of their studies, students did not know how to draw. The failure of the geometric method was a subject of discussion at Chicago's Exposition in 1894. Simultaneous with the failure of the official model, an exhibition containing drawings by eight-year-old students from an American school that refused to follow the geometric model surprised everyone and was considered the starting point for an education reform led by M. J. Liberty Tadd, resulting in the promotion of the direct study of drawing *au naturel* (Frizzarini et al, 2015, p. 48-49, our translation).

¹¹ *Revista de Ensino* was published from 1902 to 1919. It was an initiative of Associação Beneficente do Professorado Público de São Paulo (São Paulo Public Teachers' Charity, in free translation) (Shieh, 2010).

Researchers Renaud D'Enfert and Daniel Lagoutte (2004) highlighted the importance of the universal expositions to the progress of foreign nations and to the vulgarization of drawing teaching. Beguery (2003) endorses D'Enfert and Lagoutte's understanding and emphasizes the effervescence of universal expositions gave rise to discussions and debates on the purpose of drawing. More specifically, London's Great Exhibition (1851) presented the creation of institutions like South Kensington Museum, as well as the comparison of drawings produced by children from different countries.

Revista de Ensino also published Brazilians' positions advocating the introduction of the so-called "natural drawing", which proposed to abandon the drawing of geometric figures and to replace it with the drawing of natural objects that were familiar to students, as in the extract below:

All methods have produced completely ineffective results so far. It is exclusively imbued with geometric figures (referring to the geometric method) which definitely do not educate the hand or the sight. [...] We need a method through which it is possible to draw any object presented to us – it is natural drawing. It must obey the hand and the thoughts heartily. (Canto, 1906, p. 768, our translation).

In the state of São Paulo, natural drawing was introduced in the 1905 program for the Drawing subject, which made geometric drawing lose space. As for the Geometry subject, constructions with ruler and compass were introduced in the 1918 program. Both natural drawing and geometric drawing were kept in the programs until 1949/50.

In general, even without a careful analysis of all Brazilian pedagogical programs, the partial results show that natural drawing was inserted into Brazilian primary school, but geometric drawing and constructions with ruler and compass were not totally abandoned. Both proposals were promoted in normative resolutions, sometimes in separate subjects, sometimes in the same subject.

What actually lost prominence both in textbooks and in normative instructions was freehand drawing of geometric figures, which virtually disappeared. The designation "practical geometry" started to be seen as the practice of drawing geometric figures with the support of ruler and compass, that is to say, a practice linked to the use of instruments.

Handwork

The subject *Trabalhos Manuais*¹² (Handwork, in free translation), was first introduced in Rio de Janeiro, the Federal District, in 1890, along with the principles

¹² *Handwork*, with the capital H, will be used to refer to the school subject and *handwork*, with no capital H, will refer to all kinds of manual work that do not refer to the name of the discipline.

of the intuitive method. In the state of São Paulo, the subject was created a short time afterwards, in 1894, and then it was expanded to other Brazilian states¹³.

In 1891, a Brazilian teacher – Manoel Frazão¹⁴ – traveled to Europe on a pedagogical mission¹⁵, visited Nääs School in Sweden and took courses on how to “teach handwork”. In his report, he dedicated a special chapter to the worldwide trend towards teaching handwork. Frazão (1893) wrote that Nääs was considered the cradle of educational handwork with Fröbel’s¹⁶ proposal.

Frazão (1893) reported that he attended courses at Nääs School with teachers from several European countries, as well as from North America, Russia and Africa. Such information supports what Matasci (2015) calls the first globalization by identifying the importance of international contacts in the construction of the republican education systems in the 19th century, both in France and in other countries, too.

Another traveler, teacher Luiz Reis¹⁷, described his pedagogical trip to Belgium in a report in which he discussed the geometry program for the early years of primary school and instructions for teachers, signaling a strong link among geometry, drawing and handwork. He described six steps teachers should follow in order to teach geometric shapes, and in all of them it is possible to identify a close connection to the observation and analysis of objects that were previously made, like Fröbel’s toys, so as to highlight intuitive and practical aspects supported by drawings and handwork, which played an important role in demonstrating and justifying geometric properties and theorems.

Fröbel’s ideas had been circulating in Brazil in the first Jardim de Infância (Kindergarten, in free translation) in Rio de Janeiro in 1875 before the pedagogical missions. Joaquim José Menezes Vieira¹⁸ and his wife were the headmasters of Jardim de Infância. In 1882, Menezes de Vieira published *Manual para Jardim de Infância: ligeira compilação*¹⁹ (Kindergarten Manual: A Quick Compilation, in free

¹³ Besides the cultural diversity existing in Brazilian states, the states had autonomy to promote their own school programs. In Brazil, at the time analyzed here, there was not a national teaching program.

¹⁴ Manoel José Pereira Frazão graduated from Academia Militar do Rio de Janeiro. He was a primary teacher and school headmaster (Conceição, 2019).

¹⁵ Missions or pedagogical trips were commonplace at the end of the 19th century because it was a way through which a country could access educational updates. With the first globalization, connections between countries were intensified and had direct effects on education and social life (Conceição, 2019).

¹⁶ Friedrich Wilhelm August Fröbel (1782-1852), a German educator, founded the first Kindergarten and a toy factory. He was Pestalozzi’s intern and started employing handwork through spontaneous play, thus creating a series of exercises based on “gifts”.

¹⁷ Luiz Augusto Reis was a primary teacher and school headmaster (Conceição, 2019).

¹⁸ Joaquim José de Menezes Vieira was a doctor and educator. He founded and directed Colégio Menezes. He took part in Paris Exposition (1889).

¹⁹ The textbook was originally published in 1882, but the edition analyzed in this paper was published by historian Maria Helena Camara Bastos in 2011.

translation), in which he presented Fröbel's gifts (toys) in detail, explained their purpose and proposed activities that teachers should perform using the material. It was a genuine course for teachers, step by step, class by class.

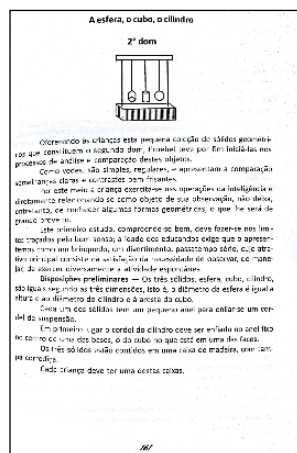
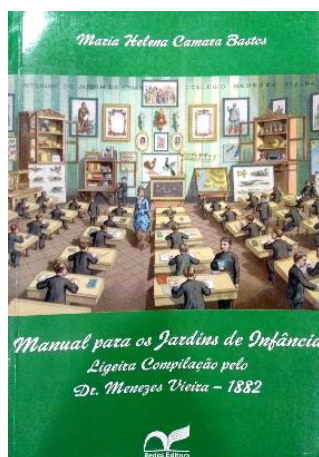


Figure 4 – Cover of Bastos (2011) and Bastos (2011, p. 161).

Bastos' textbook (2011) does not contain references, so it is not possible to know what Menezes Vieira based on to compile Fröbel's proposal, but it encompasses 7 gifts: (1) The ball, (2) The sphere, the cube, the cylinder; (3) The cube divided into eight cubes; (4) The prisms; (5) The cube divided into 27 cubes; (6) The cube divided into 27 bricks; (7) The little wooden boards. Each gift is accompanied by a set of exercises. For instance, in the 1st gift there are ten exercises that include: the distribution of balls, the shape, positions in space, the relative positions of two objects, movement, the direction of movement, new relationships, the study of the characteristics of the object, weight, lines and their directions.

About the travelers' written reports, while visiting different countries, all of them offered information on primary school teaching programs. Conceição's thesis (2019) compiled and analyzed programs from different countries, and summarized them in Table 1.

One of Conceição's analyses (2019) identified the presence of the Drawing subject in all programs from the visited countries, which was in line with the analysis performed in Brazil: geometry and drawing study were closely linked, first through freehand drawing and later, at the end of the 19th century, through geometric drawing with ruler and compass.

Another observation stemming from Table 1 is that the Handwork subject was present in almost all countries, except Portugal and Italy. Finally, something that draws our attention is the absence of geometry as a subject in the first two grades. As a rule of thumb, it was inserted in the final years. On the other hand, geometric concepts were inserted into other subjects, especially drawing and handwork

practice. It seems that in order to introduce the teaching of geometry, it was necessary to integrate it into other types of knowledge.

Table 1: Primary School Subjects reported by Brazilian travelers

Country	Subjects	Geometry
Italy	Reading, Accounting, Writing, Notions of Things, The Metric System, Drawing	Inserted in the grades after elementary school
Switzerland	Mother Tongue, Arithmetic, German Language, Geometry , Geography, History, Drawing , Calligraphy, Gymnastics and Games, Singing, Handwork	From the third year on
England	Reading, Writing, Accounting, Drawing , Handwork	Each school decided if Geometry should be offered or not
Sweden	Religion and Moral, National Language, Writing, Arithmetic, Geography, History, Slodj , Singing and Vocal Music, Drawing , Gymnastics	Geometry was taught in the two final years
Belgium	Reading, Writing, Calculations (Mathematics), The Metric System, Elements of the French Language, Elements of the Flemish Language, Geography, History, Elements of Drawing , Singing, Gymnastics, Handwork , Geometric Shapes	
France	Moral and Civic Instruction, Reading and Writing, National Language, Calculations and The Metric System, History and Geography, Object Lessons, Elements of Drawing , Handwork , Gymnastics	Geometry appeared in superior primary instruction
Portugal	Reading, Writing, The Four Operations with Integers and Fractions, Elements of Portuguese Grammar, Principles of the Decimal Metric System, Principles of Drawing , Christian Doctrine	
Spain	Reading and Writing, Arithmetic, Spanish Language, The Metric System, Drawing , Handwork , History and Geography, Gymnastics	

In the state of São Paulo, the Handwork subject was present in all programs from 1894 to 1949/50, for all school years. From 1894 to 1925, it proposed concrete activities referring to solids and plane figures. After 1925, proposals encompassing modelling, cardboard work, cutting and paper folding also became a part of the Geometry subject.

Frizzarini's thesis (2018) identified two different movements while analyzing the relationships established between the Handwork subject in the states of São Paulo and Rio de Janeiro and mathematical knowledge. The first movement (end of the 19th century and beginning of the 20th century), characterized by the intuitive method, was based on copying good models, so students could learn by observing things and objects. Handwork activities were isolated in a specific subject, which used the concepts of measurements, angles and geometric shapes, but there was no intention to support the teaching of such concepts: they were only harnessed through copying.

Moreover, another articulation was identified in the second movement, which started in the 1920s supported by the pedagogical postulates of New Education, and which considered children the promoters of teaching, so school activities should be based on children's interests. Handwork started to be inserted into other subjects as a teaching methodology, especially into subjects that encompassed shapes, drawing, arithmetic and geometry. Seemingly, the proximity and integration of handwork into geometry teaching took a longer time to happen in Brazil.

Tachymetry

Tachymetry was a geometry teaching method created by Edouard Lagout (1820-1885), a French engineer, in order to reduce course time for construction professionals, with strong visual appeal and in a fast way, thus justifying the Greek prefix *tachy*, which means fast, that which functions in a fast way. The tachymetric method had its origins linked to professional formation, but over time the characteristics highlighted by the method, especially speed and the practicality associated with its principles, led the method to be transposed and adapted to primary school by Jules Dalsème²⁰, and it arrived in Brazil later (Leme da Silva & Moyon, 2020).

In the textbook *Panorama de la Géométrie Tachy-métrie – Géométrie em trois leçons*, Lagout (1872) listed ten elements to characterize his method: (1) Reform of words, (2) Reform of Sterile Truths, (3) Aesthetic Reform of Drawing, (4) Indicative Reform of Drawing, (5) Reform of Teaching by Hand or Geometric Manipulations, (6) Mnemonic Reform, (7) Reform of Continuous Syllogism, (8) Reform of All Equivalence Proofs, (9) Reform of the Fundamental Basis and (10) Reform of Artificial Division and of the Complex Classification of Truths.

Although the tachymetric method was developed for the purpose of forming construction professionals, it was possible to identify the similarities it had in relation to the intuitive method, especially principles: emphasis on concept terminology should not be prioritized, the relevance of the visual aspect, the use of colors to contrast figures, beautiful drawings, the importance of manipulating didactic materials by hand.

²⁰ Mathematics teacher at *École Normale des Instituteurs de la Seine* in France.

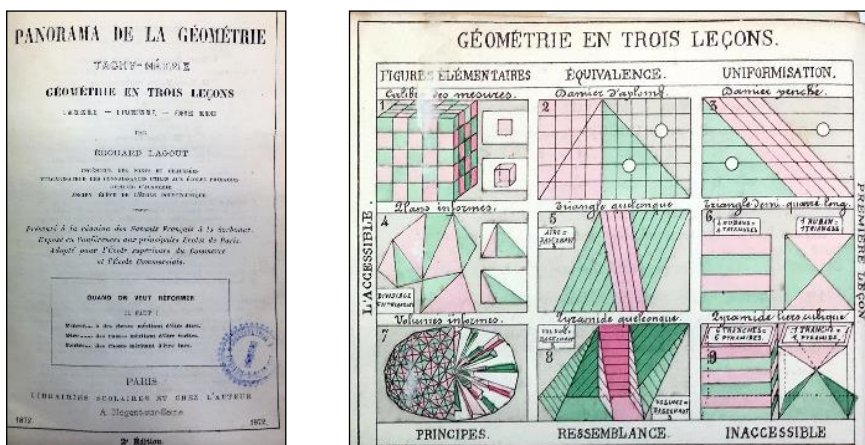


Figure 5 – Cover of Lagout (1872) and illustrations from the textbook (p.23).

The adaptation and dissemination of the tachymetric method for primary school was led by Jules Dalsème, who published the textbook *Premières Notions de Takymétrie (Géométrie Naturelle)* in 1880. It was an adaptation of Lagout's proposal that had primary teachers as its target audience, highlighting pedagogical aspects for teaching children. Dalsème (1880) presented and defined the main concepts that would be used throughout his textbook and explained to readers what Takymétrie was: "Tachymetry teaches us to measure lines, surfaces and volumes by using simple and fair rules" (1880, p. 5, our translation). Dalsème's textbook can be considered a didactical version of Lagout's proposal.

About the circulation of Lagout's and Dalsème's textbooks in Brazil, we know that Edouard Lagout's textbook *Takitechnie – Baccalauréat ès-sciences – A Livre Ouvert* (1881) circulated in Rio de Janeiro and São Paulo, while Dalsème's publication *Premières Notions de Takymétrie (Géométrie Naturelle) – à l'usage des Écoles Primaires* (1880) was found at Escola Normal de São Paulo's collection. Nevertheless, none of them was translated into Portuguese.

Another relevant document to Brazilian education reforms, Rui Barbosa's report, referred to tachymetry. He argued that:

Tachymetry is the concretization of geometry. It is the teaching of geometry through material evidence, the accommodation of geometry to the most rudimentary intelligences: it is object lessons applied to the measurement of extensions and volumes. [...] the tachymetric method is, therefore, the most rigorous, the most solid, the most feasible adaptation of intuitive pedagogy laws to popular geometry teaching, to children's geometric instruction. (Barbosa, 1947, p. 290-292, our translation).

In Brazilian textbooks, we identified the appropriation of tachymetry as a geometry teaching method. It was identified in the textbook *Noções Intuitivas de Geometria Elementar* (Elementary Geometry Intuitive Notions, in free translation)

by Gabriel Prestes²¹, published in 1895. The textbook is a guide to primary teachers who aimed to have practical instructions on geometry teaching. In the preface, Prestes wrote that he acknowledged Lagout and Dalsème as the creators of the tachymetric system, according to which it was possible to determine the measurements of lines, surfaces and volumes without a detailed study of geometric properties. However, he said that he had not had the opportunity to examine their textbooks. Prestes, then, used another French textbook as a reference to support his work: *Premiers Éléments de Géométrie Expérimentale* by Paul Bert.

The organization of Prestes' proposal (1895) broke with traditional geometry teaching, especially concerning the way measurements were highlighted. According to Prestes, students were able to recognize the several types of magnitude whose measurement constitutes the object of geometry: they did not find it difficult to differentiate a line, a sheet of paper, a glass. Geometry, in turn, had the purpose of measuring said magnitudes: lines, surfaces and volumes. In summary, what was in question was the insertion of the study of magnitudes and their respective measurements as an elementary starting point for geometry teaching. The point was to place such knowledge – magnitudes and measurements – at the beginning of teaching as a basis for future studies.

Prestes (1895) suggested starting the 2nd year with the study of direct measurements, which corresponded to choosing a unit and comparing that unit practically with what one desired to measure. It meant measuring intuitively, specifying how many times the chosen unit fit the object to be measured, and adopting the same procedure to measure length, area and volume.

In 1896, Prestes published the sequel to his textbook in an article on the journal *A Escola Publica*²² from São Paulo. In the article, he gave suggestions related to teaching 3rd-year students – they should learn indirect measurements –, which corresponded to using formulas to calculate areas and volumes. However, there was concern about detailing equivalences so that formulas could be understood.

Gabriel Prestes' proposal considered measurements (both direct and indirect) as the central point of geometry teaching, starting from the 2nd year of primary school. Moreover, he proposed the study of all magnitudes together: length, surface and volume. His proposal did not highlight speed in geometry teaching – the prefix *tachy* was not even mentioned – but the organization of a study in which measurements gained special prominence and that followed a pedagogical sequence. It started with indirect measurements, considered intuitive, concrete and practical: students measured objects built by themselves. Only later there was the systematization of measurements, which corresponded to indirect measurements, as an abstract process of synthesis in which the formula used in the calculation

²¹ Gabriel Prestes graduated from Escola Normal de São Paulo. In 1893, he became Escola Normal's headmaster. Prestes' administration (1893-1898) was the heyday of public instruction in São Paulo (Monarcha, 1999).

²² The journal *A Escola Publica* circulated in São Paulo from 1893 to 1897, with a section called *Pratic Pedagogy*.

made it much faster. Nevertheless, the introduction of the formulas should happen through explanations based on drawings and comparisons; it should lead students' reasoning so that the formula could acquire meaning.

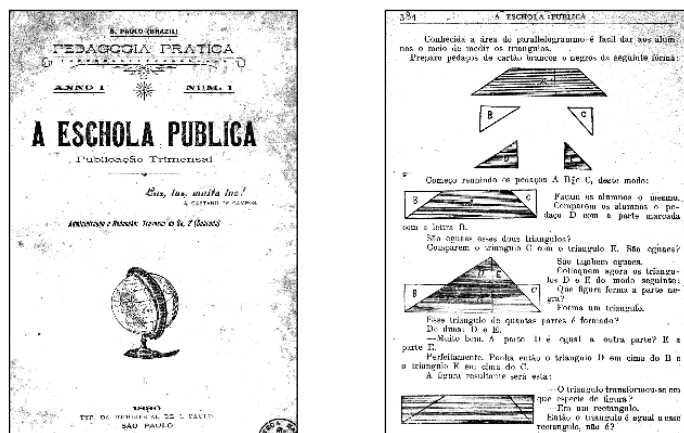


Figure 6 – Cover of A Escola Publica (1896) and Prestes (1896, p. 384)

In the São Paulo pedagogical programs, the term tachymetry was mentioned in the 1892 program, then only in the 1933 program, always linking tachymetry to “practical geometry” applied to the measurement of areas and volumes. However, Prestes’ proposal – that length, area and volume should be introduced in the 2nd year through direct measurement; indirect measurement should appear later – did not gain prominence in the programs.

In summary, the Brazilian appropriations of the tachymetric method were seemingly linked to the calculation of areas and volumes through formulas, with the caveat that they should be explained to students, who were expected to understand how formulas were obtained, in a similar way to the principles of demonstration of equivalence proposed by Lagout (1872).

Conclusion

The three examples analyzed offer evidence that the configuration of proposals for geometry teaching in Brazilian primary school in the 19th century and at the beginning of the 20th century was closely linked to foreign proposals, programs and trends. The three categories – drawing, handwork and tachymetry – that were selected for the analysis of Brazilian appropriations stemming from foreign contexts proved to be fundamental to understand proposals for geometry teaching in Brazilian primary education. Seemingly, school geometry was fostered, collated and developed in direct connection with other subjects, knowledge or teaching methods²³.

²³ The complete results of this study can be found in Leme da Silva (2021).

The history of Brazilian primary teaching seems to have its own particularities in comparison with secondary teaching, in the sense that the proposed teaching methods (freehand drawing, drawing with rulers and compasses, manipulation of concrete materials or based on fast calculations of measurements) ended up configuring different types of school geometry. It seems to be crucial to comprehend pedagogical trends, like the intuitive method and the need to link teaching to a practical activity, and even the tachymetric method in order to understand the transformations that happened in geometry teaching proposals over time. Advocating a type of school geometry that is intuitive, practical and experimental – something present even nowadays – could be interpreted as a legacy from past proposals.

Furthermore, it is equally relevant to highlight that the production of school geometry was a result of exchanges among several authors, through a constant process of interchange, readings and interpretations of national and international experiences, which reiterates the importance of pedagogical knowledge circulation (Fontaine, 2022). It was possible to identify the dynamic and collective aspect that characterized the production process of school geometry, which, albeit linked to the designation of a “practical” type of geometry, undergoes transformations and carries different meanings over time.

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