From 'stelkunst' to 'which algebra?' Changes in Dutch school algebra

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Abstract

In the Netherlands, from 1863 on, schools for secondary education in which mathematics, sciences and other subjects were taught to all pupils, were established in several towns. At that time algebra already was an art which was relatively wide-spread among Dutch mathematical practitioners and teachers of mathematics. Textbooks in Dutch, but also in English, French and German and journals were available to provide knowledge about algebraic techniques, simple exercises and problems. Thus when in 1870 a national mathematics curriculum was formulated, ideas about the elementary algebra to be taught in lower secondary school were reasonably well established. However, what algebra topics should be taught in higher secondary was less clear. During the next 150 years there were about seven relatively influential changes in the mathematics curriculum. In this paper the reasons for and effects on the algebra programme of the first three of these changes will be discussed.

Keywords: school algebra, HBS, aims of mathematics education, formative value, linear algebra

Introduction

What is algebra? The well-known definition by Euler was valid for a long time.

> Algebra is thus also described as the art in which is shown how to determine an unknown quantity from known quantities (Euler, 1771, 2nd volume).[1]

Around the start of the 20th century the drive towards more abstraction and the study of structures resulted in two different views on algebra, as is clear from the definitions of algebra in Merriam-Webster (2022):

> 1) A generalization of arithmetic in which letters representing numbers are combined according to the rules of arithmetic.
> 2) Any of various systems or branches of mathematics or logic concerned with the properties and relationships of abstract entities (such as complex numbers, matrices, sets, vectors, groups, rings, or fields) manipulated in symbolic form under operations often analogous to those of arithmetic.

The Brittannica (2022) calls the second form *modern* algebra, reminiscent of the early 20th century.

School algebra usually has more to do with the description by Euler, with generalization of arithmetic, with the relations between physical objects and the solution of equations (Katz & Hunger-Parshall, 2014), though during the 1960's there were attempts to introduce abstract modern algebra into the curriculum. Algebra courses at universities followed the development towards more abstraction and ultimately the study of structures rather than physical objects, as

[1]All translations are by the author.

Bjarnadóttir, K., Furinghetti, F., Karp, A., Prytz, J., Schubring, G., Weiss, Y., & Zender, J. (Eds.) (2023). *"Dig where you stand" 7. Proceedings of the seventh International Conference on the History of Mathematics Education.* Münster: WTM-Verlag.

exemplified by Van der Waerden's textbooks[2]. Named *Modern Algebra* in 1930, the title became plain *Algebra* with the fourth edition in 1955. Abstract algebra had become mainstream, even for first-year students. What changes occurred in school algebra during the same period?

Which changes in views, aims and content do we find in school algebra in secondary education in the Netherlands during the first hundred years of its existence? Which influences and influencers are apparent?

Though in the Netherlands algebra was taught from the early seventeenth century, the starting year of mathematics, including algebra, as a school discipline[3] is taken as 1863. In that year the Law on Secondary Education was accepted by Parliament (Krüger, 2014). As a consequence in several towns a HBS (Hogere Burger School or Higher Burgher School) was established. The HBS was meant for the sons of well-to-do citizens who would pursue a career in industry or administration. An important role for the HBS was preparation for the Polytechnic School in Delft. The programme of the HBS encompassed 18 subjects, of which mathematics, sciences and modern languages formed the largest part (Krüger, 2014; Krüger, 2016). It is important to note that until 1917 a HBS certificate did not provide the right to sit university examinations; one needed a gymnasium certificate or provide evidence of knowledge of Latin and Greek.

The first formal examination syllabus dates from 1870. During the next 150 years there were about seven relatively influential changes in the mathematics curriculum for secondary education, in 1937, 1958, 1968, 1985/1990, 1998, 2007 and 2015. In this paper the reasons for the changes, as formulated in the aims of the curricula and the consequences (if any) of these changes for school algebra are explored for the period 1870–1970. The period 1970–2015 may be discussed in a follow-up paper.

School algebra

Kenbir, Clements & Ellerton (2018) draw attention to the various views on school algebra in the past and present. School algebra may be seen as the study of generalized arithmetic and as a body of knowledge which prepares students for higher mathematical and scientific studies as well as a gatekeeper for entry to higher studies. School algebra may also be seen as an integral component of mathematical modelling of real-world contexts and as a vehicle for generalizing numerical, geometrical and other mathematical structures or, more abstract, as the study of variables — symbols which can represent different quantities and display

[2]Bartel L. van der Waerden (1903–1996) was a student of Emmy Noether, he became professor of mathematics in Groningen and Zürich. In 1930 he published *Moderne Algebra*. The two volumes went through several editions and were translated into several languages.

[3]School discipline meaning a subject that is taught, both for its content and as intellectual stimulus, which has to a certain extent an autonomous development and is linked with didactics and pedagogy (Chervel, 1988; Cardon-Quint & d'Enfert, 2017).

relationships between those quantities. Any number of these viewpoints could implicitly or explicitly be represented in curricula.

In Europe the teaching and learning of symbolic algebra became more common in the 17th century, when the first text books treating algebra appeared and algebra became a teaching subject in some educational institutes. Ponte & Guamares (2014) give a historical overview of the teaching of algebra in some European countries. Symbolic algebra became the preferred method to research and teach polynomials and find solutions to polynomial equations, facilitated by the acceptance of Cartesian notation. With the increase in institutes for primary, secondary and higher education, school mathematics developed gradually as an autonomous discipline, with algebra as a sub-discipline.

Maybe the first author to publish an algebra textbook, using Cartesian notation throughout, was the Dutchman Gerard Kinckhuysen, with *Algebra ofte Stel-konst,* published in 1661. The year before he already used Cartesian notation in his treatment of conic curves in *De Grondt der Meet-konst* [The principles of the Art of Measuring] In France, Antoine Arnauld published in 1667 *Nouveaux Élémens de de Géométrie*, in which book he started with the development of algebra. Jean Prestet published *Éléments des Mathématiques* in 1675, dedicated to algebra. Both books were used in educational institutes (Ponte & Guamares, 2014). In Portugal in 1772, following the expulsion of the Jesuits in 1759, the reform of the Coimbra University entailed the establishment of a Mathematics Faculty with four mathematics courses, one of which was algebra and infinitesimal calculus (Mota, 2011).

In the Netherlands Kinckhuysen's book was followed by many textbooks on algebra, by various authors; these books were meant for those who studied algebra, either with a tutor, in small private schools or through self-instruction (Krüger, 2016). From the early 17th century algebra was taught at universities in undergraduate courses, in engineering courses, naval schools, military schools, extended primary schools (French schools), institutes for technical education and by private tutors.

Recently, Smid (2022) published a thorough historiography of Dutch mathematics education from 1800–2000. From the early 19th century, Dutch education became more institutionalized and algebra was taught at an increasing number of gymnasia and (extended) primary schools; from then on, textbooks were published specifically for use in those schools (Beckers, 2003; Krüger, 2014). In 1863 at the start of Dutch secondary education, algebra was primarily seen as generalized arithmetic, a preparation for the Polytechnic School and perhaps to some extent as a way to generalize mathematical structures.

Before turning to the history of Dutch school algebra it is worthwhile to realize that in the teaching of mathematics and specifically the teaching of algebra several issues crop up, at least since the 18th century (Drijvers et al., 2011; Tirosh & Tsamir, 2008). They occasionally led to bitter disputes, especially in the 20th and early 21st century, also in the Netherlands. Examples of topics for disagreement are:

the level of rigour or the relation between intuition and rigour
flexible analytical reasoning skills versus procedural skills
'pure' algebra versus algebra which is useful in practices outside mathematics
the topics to include in school algebra
the gap between school algebra and requirements of university, more specifically technical universities
didactic methods to be used

The HBS and the algebra programme from 1863–1937

At the start of the HBS the main actors with regards to the mathematics programme were the prime minister, J. R. Thorbecke, the civil servant and inspector of secondary education, D. J. Steyn Parvé and the nathenatics teachers . The aims of the mathematics curriculum varied somewhat, depending on who was talking; they may be summarized as follows (Krüger, 2014, pp. 383–402):

contribution to general education
preparation for the Polytechnic School and for entrance exams of other technical studies (military, marine, veterinary, etc.)
support of the instruction in physics and other sciences
development of a specific way of thinking and reasoning, logical thinking, the 'formative value'.

While occasionally doubts were expressed about the value of some parts of the mathematics curriculum, such as descriptive geometry obligatory for all students, no such doubts were noticeable about the applicability and formative value of algebra. However, the opinions varied about which algebraic topics should be taught. As we see below, during the first decennia of the HBS the choice of topics taught in the algebra course of the years four and five depended to some extent on the teacher. In the first half of the twentieth century, new topics in the syllabus, such as differentiation and integration, also came under the heading of algebra.

Initially there was no prescribed content for the subjects taught at the HBS, nor was there a syllabus for the final examinations at the end of the fifth year. According to prime minister Thorbecke, the purpose of mathematics at the HBS was to support physics and to prepare for the Polytechnis School (and similar institutes). For algebra he mentioned equations up to and including second degree, arithmetical and geometrical series and the binomial theorem (Krüger, 2014, pp. 246). Though the first final examinations took place in 1866, with 23 candidates, regulations and a syllabus had to wait until 1870. In this first syllabus it was emphasized that merely knowledge of the main topics in each of the subjects should be examined and that only the limits of that knowledge were indicated in the syllabus and not the details. Proofs of theorems and lengthy calculations were to be omitted and the use of logarithms wherever possible was recommended.

There would be four separate exams: algebra (with arithmetic), geometry, trigonometry and descriptive geometry.

See table 1 for the algebra topics in the syllabus.

Table 1. HBS examination syllabus of arithmetic and algebra in 1870

topics	skills
Logarithms, use of Arithmetic and geometric series Principles of power series Indeterminate equations Exponential equations Binomial theorem	To choose the right method to solve problems. To determine the answer with the correct precision.

Note that first and second degree equations were not mentioned, as these were taught in the first three years. The programme was more extensive than the advice of prime minister Thorbecke.

There was general agreement on the algebra programme in lower secondary school. In all textbooks of the 19th century one finds explanation of terms and symbols, the four basic operations with algebraic forms, including fractions, procedures with roots (quite elaborate), powers of algebraic forms, fractional and negative exponents and equations of first and second degree with one or more unknowns (Krüger, 2014; Smid, 2022). The content of algebra in year four and five was a different matter. School programmes and textbooks show the existence of differences between schools. The choice of topics was influenced by the background and ambition of the mathematics teacher(s), what they saw as aims of mathematics education and the requirements of entrance exams for further education, such as the entrance exams of military and naval schools. Initially the final exams had less influence than at present, as before the 1890's the majority of students left school before the final (fifth) year, to continue at another institute, such as military or naval schools, to start as teacher trainee or to enter the workforce in another capacity (Krüger, 2014, chapter IV, 5.3.3).

Apart from the topics mentioned in the syllabus, one finds in school programmes and textbooks topics such as permutations and combinations, probability theory, complex numbers, harmonic numbers, harmonic series and functions. During the first decennia at least 20 different algebra books were in use at about 40 Higher Burgher Schools, either existing books for extended primary schools, the Royal Military Academy, Naval schools, etc. or newly written by teachers (Krüger, 2014, pp.510–511). The relative autonomy and the professionalism of the teachers, together with the limited level of regulation were important factors for this variety in textbooks and topics.

The growing view of mathematics as a rigorous axiomatic system, an independent field of research, influenced ideas of teachers and textbook authors about the aims of school mathematics, the content and didactics of school algebra and the rigorousness of treatment (Krüger 2014, ch. IV, par. 5.6.2). During the first half of the 20th century a trend in the direction of pure mathematics was noticeable; applicability was frowned upon by influential authors, such as E. J. Dijksterhuis. Initially other views made themselves known. In the early 20th century attempts to modernize the programme through the introduction of calculus and functions in the algebra curriculum came from two HBS-teachers, both with an engineering degree. Influenced by Felix Klein, F. J. Vaes and C. A. Cikot proposed the introduction of differential and integral calculus in the curriculum. They emphasized the applicability in sciences and technology, the expected improved coherence in education and mentioned that these topics were already implicitly used in physics and mechanics. Their attempt failed, for various reasons (Smid, 2022).

The discussion about the mathematics programme of the HBS, the aims, the content, the overload and the quality of the examinations continued. E.J. Dijksterhuis, from 1916 teacher at a HBS, later in life a well-known historian of science, advocated, the teaching of mathematics as a strictly axiomatic system. In 1925 the Inspectorate of secondary education asked a committee, with H.J.E. Beth, mathematics teacher and director of a HBS as chairman and E.J. Dijksterhuis as secretary, to research the state of mathematics at the HBS and to formulate proposals for improvement. Both Beth and Dijksterhuid had university degrees in mathematics. In 1926 the Beth-Dijksterhuis committee indeed recommended introduction of differential and integral calculus in the curriculum of the HBS, but primarily because of its formative value and as part of the cultural development, so with an aim that was very different of the ideas of Vaes and Cikot. The committee wrote that algebra education ought to stimulate the development of 'functional thinking', which meant emphasizing the central position of functions. (*Euclides* 2(4), pp. 113–139). On the aims of mathematics education the committee stated that

> Mathematics education aims primarily at formation of the mind and cultural development, secondarily at installing useful knowledge.

However, the committee realized also that in the choice of topics to reach this primary aim, one ought to keep in mind the interest of those pupils for whom the HBS meant a preparation for further study (such as engineering studies).

The unsatisfactory results of the final examinations, were variously and contradictory ascribed to: insufficient time to treat the topics properly, too few and too easy topics in algebra, lack of intelligence of the pupils, the vagueness of the examination programme and the varying quality of the examination questions. A change in the programme of algebra would thus be necessary. A small step was taken in 1931, when questions on a limited concept of functions and their graphic representations were permitted in the final exams, with little success (*Euclides* 9(1), pp. 11–22; *Euclides* 9(5), p. 225, *Euclides* 12(6), pp. 283–300). Nothing else changed until 1937, though Beth wrote already in 1930:

> That the [examination] questions for algebra are in general poor, has nothing to do with uniformity of the examination, in my opinion. The cause is simply insufficient and too simple topics (*Euclides* 07(1–2), pp. 64–65).

At the time the algebra topics occurring in the exam programme were series (arithmetical and geometrical), logarithms, compound interest, equations of first and second degree with one or more unknowns and problems which may be phrased in terms of such equations. According to Beth, these topics were treated by the end of the third year.

HBS 1937: a very restrained renovation of the curriculum, but not of the exam syllabus

Reasons for changes in the curriculum were thus the need for adjustment to the developments in algebra and the necessity of a general improvement of the mathematics programme. Some topics, such as permutations and combinations, disappeared quietly from the textbooks, however, as the autonomy of teachers was respected, the books still were rather full, with new topics squeezed in.

The Beth-Dijksterhuis committee was dominated by men with an outspoken view on aims of mathematics education and on algebra within mathematics. They feared that the teaching of pure mathematics with its logical structure and strict demands would disappear, due to the increase in material welfare and popularisation of sciences. In 1937 Beth formulated once more in *Euclides*, a journal for mathematics teachers, the aims of mathematics at the HBS as follows.

> She[4] [the committee] wishes primarily to keep in mind the formative value of the practice of mathematics and only secondarily to pay attention to the practical use that some parts of mathematics could have later on for part of her students; she thus considers installing of fundamental theoretical insights more important than the development of technical skills (*Euclides* 13(6), p. 271).

Compared to the aims mentioned at the start of the HBS, this seems a curious narrow vision on mathematics education in secondary schools, with the supposed formative value of mathematical training enlarged enormously and in prime place. For lower secondary the programme included emphasis on the concept of number

In 1937 the new proposed curriculum for the years three to five mentioned for algebra: equations of first and second degree, irrational and complex numbers, linear and quadratic functions, pencil of lines, orthogonal hyperbola, cubic functions, the remainder theorem, arithmetic and geometric series, sequences, logarithms, limits, differentiation and integration (Kiers, 1947; *Euclides* 13(6), pp. 270–282): Compound interest, logarithmic and exponential equations had become optional, but it was nevertheless an ambitious programme, with a lot of topics under the umbrella of algebra. However, as the exam syllabus did not change at all, not much will have happened. Why were similar changes not made to the syllabus? That was partly to allow the teachers sufficient time to get accustomed to teaching the new topics.

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[4]Beth used the feminine form here.

A few years later WWII engulfed Europe and a major part of the rest of the world. There were other worries than an outdated examination syllabus.

HBS 1958: new convictions, a new programme and a new syllabus

Some years after WWII a strong drive to modernize mathematics education became apparent in many countries. In the Netherlands new actors, with a more modern outlook and interest in what happened in mathematics education outside their own country, became active. Some of the older generation remained, but with negligible influence. The curriculum of the gymnasium dated from 1919, the curriculum of the HBS was more recent (1937), but not linked with the examination syllabus. Attempts to improve existing mathematics curricula of HBS and gymnasium or develop and combine new ones were undertaken by at least three groups: Wimecos, the association of teachers in mathematics, mechanics and cosmography at the HBS, Liwenagel, the association of teachers at the gymnasia and the Wiskunde Werkgroep [Mathematics Working Group], with Hans Freudenthal as an influential member (la Bastide-van Gemert, 2015, pp. 148–155). In 1952 the Wiskunde Werkgroep produced a concept curriculum for both HBS and gymnasium, which was not officially accepted, but served as a basis for the next and more successful attempt. In 1954, encouraged by the Inspectorate, Wimecos formed a Committee that, with support of Liwenagel and using the concept programme of the Wiskunde Werkgroep, produced a communal concept curriculum and exam syllabus for HBS and gymnasium (Smid, 2022). The Committee consisted of the mathematics teachers C. J. Alders, board member of Wimecos, A. Holwerda, P. G. J. Vredenduin, board member of Liwenagel, J. H. Wansink, board member of Wimecos and of L. N. H. Bunt, teacher trainer at the University of Utrecht and of Groningen and a member of the Pedagogical Institute in Utrecht. Wansink, Bunt and Vredenduin were also members of the Wiskunde Werkgroep.

After WWII the journal for mathematics teachers, *Euclides*, attracted several university mathematicians as cooperator; it also published more or less regularly articles about mathematics teaching in other countries. Thus the actors involved in discussions on the mathematics programme were not only teachers, but also productive mathematicians, contrary to the situation before WWII, as described in Smid (2022).

The report of the committee was published in *Euclides* in 1955 (*Euclides* 30 (4), pp. 149-176). The reasons for this curriculum renovation, as mentioned in the report, were the discrepancy between the curriculum of 1937 and the exam programme of 1929 of the HBS and the need to modernize mathematics education at the HBS and gymnasium. In her report the committee made the following remark.

> The continuously growing importance of mathematics for society necessitates that pupils with regards to mathematics are prepared for their future role more efficiently than before

This importance of mathematics for society in general was mentioned for the first time in a formal text. This conviction resulted in the following ordered criteria for the choice of topics in the concept curriculum for the HBS, based on implicit aims for mathematics instruction.

The relevance of the applications in school disciplines and domains which are related to general aims of education.
The level of indispensability for future study.
The level of suitability to attribute to the intended mathematical training and training of thinking, provided the use of appropriate didactics.
The relevance from a cultural-historic point of view.

These criteria or aims showed some similarity to those recognized for the HBS in the mid-19th century, but they had a broader aim. Applications should be relevant to more school disciplines than physics, mathematics should prepare for various studies, not only technical, didactics was mentioned for the first time. It was a turn-around from the ideas formulated in the 1930's.

The concept curriculum was divided into content for year 1–3, for all pupils, consisting of algebra and geometry, and content for year 4-5. The Committee proposed for the beta-stream in the final years five main topics: algebra, trigonometry (including functions), analytic geometry, solid geometry (including some remains of descriptive geometry) and statistics. Analytic geometry was an expected addition, but statistics was a bit of a surprise. After a long discussion the general assembly of Wimecos accepted the proposals. The committee strongly advised, with many examples, to keep exercises simple and to omit unnecessary complicated problems. However, she also stressed the freedom of choice for teachers. The proposed exam programme consisted of four separate exams (see below), with one grade for a) and d) together and one for b) and c) together.

a) Algebra and calculus of differentiation and integration
b) Trigonometry and analytic geometry
c) Solid geometry, including oblique orthogonal projection
d) Statistics

On August 30, 1958, the ministry published a new curriculum for the HBS and gymnasium, based on the concept of the committee, with a few exceptions. Statistics and complex numbers were not included and algebra teaching should specifically pay attention to applications in physics (*Euclides* 34–3, pp 66–72, *Euclides* 35-1, pp. 17–24). A few months later the exam programmes were published.

Table 2. HBS programme for algebra in 1958

year 1–3	year 4–5
Arithmetical and algebraic operations, also with roots, first and second degree equalities, inequalities and functions, graphic representations of functions, arithmetic and geometric series, convergent series, practical exercises in arithmetic and mental arithmetic, logarithms, approximations, absolute and relative error, sets of equations, fractional and negative exponents	The functions $\frac{ax+b}{x+c}$, $\sqrt{x}$, a^x, and their graphs. The principles of differentiation and integration; extreme values; calculations of area and volumes. Attention should be paid to the applications in physics.

The committee had provided a booklet with 250 examples of the preferred types of questions and problems; there appeared articles in the journal *Euclides* with more explanations, in which also the freedom of the teacher to teach additional topics was emphasized. It was a promising programme, but drastic changes to the system of secondary education would soon happen.

End of HBS 1968: a new structure for secondary education

After WWII it was obvious that the nearly a century-old law on secondary education, based on the static division of society in lower, middle and upper class at its foundation and the implicit assumption that girls could not and should not learn sciences and mathematics, had become a barrier to economic and social development. Also in 1958 the minister of education J. Cals, together with his State Secretary A. de Waal, proposed a new law on secondary education, which law was accepted by Parliament in 1963. As the law introduced a restructuring of the whole system, covering a great many topics, it became known by its nickname: the Mammoth law. The HBS was to be abolished, as were schools for extended primary education (ulo), the domestic schools for girls ('huishoudschool') and some other types of schools. The new structure started in 1968 and consisted of three streams for general education and four streams for pre-vocational training.

General education streams are named vwo, a programme of six years; havo, with a five-year programme and mavo, with a four-year programme. In this paper we restrict ourselves to vwo and havo. Vwo prepares for entrance into universiy and consists of gymnasium and athenaeum, respectively with and without Latin and Greek. Havo prepares its pupils for higher professional education at Polytechnic Colleges, at present named Universities of Applied Sciences. In this paper the first changes in the mathematics curricula of havo and vwo are discussed.

There were more reasons than the restructuring of the system to renovate the mathematics programmes. After the successful launch in 1957 of the first Sputnik by the Soviet Union, there was in the USA and its allies a strong urge to train more mathematically qualified people. The gap between school mathematics and mathematics at universities was considered a serious problem. The conference at Royaumont in 1959, on the advisability of renovations in curricula for mathematics, resulted amongst others in an advice to provide a better preparation for studies at universities and at the same time to provide all pupils with maths skills for use in daily life. The Bourbaki group was very much present among the professional mathematicians at the conference, as were the proponents of the New Math curriculum from the USA (la Bastide-Gemert 2015, pp. 208–224). Three Dutch representatives participated in the conference: Lucas Bunt and Piet Vredenduin (both also members of the committee of Wimecos, see above) and H. Leeman, mathematics professor at the University of Amsterdam. Lucas Bunt gave a talk about his experiments with a statistics programme in gymnasium-alpha and the appropriateness of statistics as part of school mathematics (*Euclides* 35, pp. 218–229; Smid, 2022). The three were sent on advice of Hans Freudenthal, who had declined to attend, to his later regret (la Bastide-Gemert 2015, pp. 212–224). The resulting New Math movement in education, with its emphasis on abstract structures has been discussed often, we will not go into details in this paper.

In 1961 the State Secretary of Education, G. C. Stubenrouch, installed the CMLW [Committee for Modernisation of the Curriculum for Mathematics], counting 18 members, all males (*Euclides* 37 pp. 144–151). Ten of the members were professor, professor Leeman was president. Vredenduin and Bunt were members also.

The State Secretary mentioned as aims for the mathematics in general secondary education:

- to diminish the gap between science and school mathematics through introduction of new topics in havo and vwo;
- to increase the societal relevance of mathematics;
- to explore the possibility for a separate mathematics programme for mathematically highly talented pupils.

That last aim was not mentioned previously. The CMLW organised experimental lessons, in lower and upper grades, produced texts for pupils and organised in service training for mathematics teachers, about topics in modern mathematics (*Euclides* 41, pp. 104–107).

While the CLMW was still busy with experiments and their evaluation, the new structure for secondary education became reality and the new curricula, with new textbooks, were implemented. Exit HBS, welcome to havo and atheneum.

Aspects of the modern algebra from the early 20th century had entered the curriculum, but the name 'algebra' had disappeared, as the structuring of school mathematics in sub-disciplines was abolished. It is also noteworthy that the description in the examination programme was much more detailed than before, less was left to the teachers. Finally statistics had become part of the programme. However, another major change was situated in the requirements for a diploma. In order to receive a diploma of havo or vwo a pupil had to pass an exam in six (havo) or seven (vwo) subjects; the pupils were at liberty to choose the subjects for their final exam. Mathematics was no longer a mandatory subject in upper secondary education.

Did this programme fulfil the aims the State Secretary had formulated? Time would learn.

Table 3. Mathematics programmes in 1968

	mathematics programme
lower secondary, year 1–3	Intuitive introduction in geometry, learning to order mathematically, notation and semantics of set theory and logic, geometry of transformations. No separation between subdisciplines.
havo, year 4–5	Sets and elementary logic operations. Rational and trigonometric functions, graphs, equalities and inequalities. Elementary differential calculus, derivatives, chain rule, extrema. Logarithms, logarithmic and exponential functions, graphs, simple equalities and inequalities. Analytic geometry, vector geometry. Descriptive statistics. Simple probability theory.
vwo, year 4–6 Math I	Goniometric functions and graphs (sin, cos, tan, arcsin, arccos, arctan); goniometric formula's, specified; goniometric equalities and inequalities. Number e, logarithmic and exponential functions. Differentiation of specified functions; primitives, definite integral, integration by parts, improper integral. Applications; differential equations, direction field, solving simple equations. Introduction into probability theory and mathematical statistics.
vwo, year 4–6 Math II for the talented pupils, together with Math I.	Analytic geometry; vectors, inner product; set of lines, set of points. Dependence and independence of sets of vectors, determinants; orthonormal base, normal vector, normal equation. Circle and sphere, tangent line, plane. Group of translations. Linear mapping, matrices, orthogonal maps, isometry.

Discussion

At the start of formal secondary education in the Netherlands, as exemplified by the HBS from 1863, algebra was primarily seen as generalized arithmetic, useful for the sciences, in particular physics and a necessary preparation for the Polytechnic School and similar institutes. Other views, mentioned in Kenbir et al. (2018) did not become apparent until after WWII, if at all. However, one should keep in mind that roughly until the 1890's a large proportion of the pupils left school before the final year, or if they continued until year 5, did not participate in the final examinations. Passing the final exams only gradually gained importance. Moreover, after the first three years, at the end of lower secondary school, pupils could and would enter a vocational training. So a few years of education at a HBS was seen as advantageous, but obtaining the certificate after five years was not very important initially. In lower secondary the aims of algebra education were preparation for upper secondary, but also preparation for entrance exams for various vocational courses. Thus the content of the algebra programme in lower secondary became quickly more or less standardized. In upper secondary the aims of algebra instruction were preparation for polytechnic schools, support of sciences and the formative value.

The content in upper secondary expanded, as teachers emphasized different views on the value for their pupils of topics such as for example combinatorics, complex numbers or harmonic series. So different topics might be taught by various teachers as an addition to the exam syllabus, which did not change much. There was a tendency by some mathematics and also physics teachers to consider the exam syllabus as a minimum requirement. As the regulations for the final mathematics examination specified only globally geometry (solid, descriptive and new), trigonometry and arithmetic / algebra, all new topics which did not belong to geometry or trigonometry were considered algebraic.

Changes took place gradually and for different reasons. Among mathematics teachers a tendency towards a more rigorous treatment of their topic started to prevail. That might be due to developments in research mathematics, but also to the wish to improve the position of mathematics in the hierarchy of school subjects, especially with respect to physics. Mathematics was presented as a subject, important for its own characteristics, an axiomatic structure and formative value due to its rigorous reasoning. Besides, parents and pupils attached more value to the HBS certificate, resulting in more and more time spent on training for the final examinations. The changes in attitude of teachers and the increased importance of the certificate and thus of the final exams resulted inevitably in less time for topics outside the exam syllabus. The algebra programme became limited, with a strong emphasis on procedural skills. Under the influence of these changing views, the professed aims of mathematics teaching shifted from preparation for higher technical education and support of physics to formation of the mind as primary aim, to be reached through rigorous reasoning. In practice, for most teachers and pupils the primary aim would be preparation for the final examinations, which for algebra meant exercises in procedural skills. Though there were attempts by teachers to modernize the algebra programme, for example to introduce the concept of function, differentiation and integration, they were not very successful. The majority of teachers, the most influential authors, the government and the inspectorate were rather conservative at the time, or perhaps merely disinclined to make changes which would inevitably cause unrest. The HBS was considered a superior type of school, what happened outside the country seemed irrelevant. A few new topics might enter the algebra curriculum, but not the exam syllabus.

The upheaval of WWII encouraged different points of view, a more open outlook on mathematics education and its aims. Different actors, who took more notice of what happened elsewhere, became visible. Society and the country had to be rebuilt; university mathematicians expressed interest in and ideas about mathematics in secondary schools and worked together with mathematics teachers. Mathematics education should primarily be relevant for applications in other school disciplines and for future studies. Training of mathematical thinking was a third aim. The new curriculum and syllabus which were introduced in 1954 were initiated by teachers associations, but the inspectorate had a large say in the final decision on content, as had been the case before WWII. The algebra programme for upper secondary at last contained functions and their graphs, differentiation, integration and also applications, specifically for physics. It represented a reversal of the aims and content of the programme before WWII.

This curriculum was quickly overtaken by the complete overhaul of the structure of secondary education and of school mathematics, due to the international urge to catch up with technological developments and to improve the level of education of the whole population. The strict division in geometry, algebra etc. disappeared, making it easier to introduce new topics if required.

Talent had to be promoted, at the same time a much larger proportion of the age group was supposed to enter general education. As a solution a special strand of linear algebra as a formal system for talented pupils became part of the curriculum of pre-university education, as an extension of a mathematics curriculum in which an extensive range of functions, differentiation, integration, applications and statistics formed the content. However, in line with the spirit of freedom of the sixties, the pupils were at liberty to choose their own examination subjects. The idea was to get more pupils into higher education and to get more mathematically trained students.

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