Playing "As If" as a way to learn mathematics: the concept of equality in first grade in 1969

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Abstract

Playing "As If" is perhaps one of the most commonly known game structures among children. In what Piaget has called symbolic play, or imitative play, children act as doctors, mechanics, merchants, and so on. In these moments, they put all their imagination into action, a creative imagination that Vygotsky considers to be one of the superior functions of the mind.

Based on a film made in 1969 in France in a first-grade class, this question of playing "As If" for mathematical learning is explored. In this session, during a fictional game based on the merchant's game with a doll to be dressed, the students manipulate two sets of abstract symbols to (try to) construct the notion of equality in all its generality. Because the recording shows students actually in activity, this film constitutes one of the rare testimonies recorded during the modern mathematics era, a period that is sometimes decried but which, if we focus on the didactic analysis of the practices as they have come down to us, proves to be rich in new pedagogical approaches whose consequences have lasted long after the fall of this reform.

The film that is presented is part of a series intended for teacher training and is an example of a corpus of nearly one hundred reels (16 mm, with sound and color) edited between 1969 and 1972, which are currently being preserved and valorized. A large part of these documents shows classroom sessions, where active pedagogies are articulated with abstract mathematical notions but nevertheless approached in a very concrete way.

Keywords: Mathematics education, modern mathematics, equality concept, first grade

Introduction

Born of a desire to update the teaching of mathematics from kindergarten classes to university, the so-called modern mathematics reform is a high point in the history of mathematics education in France. This worldwide movement (De Bock, 2023; Kilpatrick, 2012; Moon, 1986; Vanpaemel & De Bock, 2019) is characterized in the French context by rapid and profound changes in the primary and secondary school curricula (Barbin, 2012; Gispert, 2023). Driven by the work of the Lichnerowicz Commission, named after the mathematician who chaired it from 1966 onward, the first decrees appeared in 1969-70. As Trabal (1996, p.181) notes, these new curricula, which aim to modernize the teaching of mathematics by trying to bring it more in line with what is taught at the university, are not only the fruit of the work of a commission. They are in fact the result of the convergence of innovative ideas developed within a large community of teachers and pedagogues strongly involved in associations such as APMEP (Association des Professeurs de Mathématiques de l'Enseignement Publique) but also of psychologists in the

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tradition of Piaget. The objective of the reformers was rather to promote living mathematics (Trabal, 1996, p.183), putting the students in activity, and not to focus on a set of abstract rules taught directly. The changes were intended to be profound and lasting. However, the educational context of the time very quickly gave rise to major difficulties (d'Enfert & Gispert, 2011) with the main one being linked to the inadequate mathematical skills of the teachers. The need for mass and quality continuous training became obvious. The IREMs (Institutes of Research on the Teaching of Mathematics) were created for this purpose in 1969. A real dynamic of updating the contents and methods for the teaching of mathematics was yet to be born. It was accompanied by the production of numerous written and audio-visual resources for teachers and teacher trainers. It is precisely an example of one of these audio-visual training aids that is the focus of this chapter. In what follows, we propose an analysis of a film directed by Raoul Rossi, coproduced by the Audiovisual Center of Saint Cloud and the IREM of Lyon in 1970, entitled *Égalité d'objets*. The film lasts 22 minutes and shows a mathematics session in a first-grade class (age 6-7). The class, according to the CNDP-Canopé bibliographic note, took place in October 1969, which places it at the beginning of the school year, but above all one year before the publication of the new curriculum of 1970. The involvement of the IREM of Lyon suggests that this was an experiment that, by anticipating the reform, provided a resource for teachers. The video thus shows a teacher and her students in a school teaching context that is meant to be ordinary even if, as we see, many elements make this session pedagogically innovative. This training film on the notion of equality can be considered as a witness of the intentions of its authors but also of the changes brought about by the reform, which is certainly often criticized, but whose posterity is undeniable on the practices of mathematics teaching up to now.

The modern mathematics and the epistemological and didactic issues of the notion of equality

The reading of the French curriculum for the first grade (age 6-7) in 1970 shows that equality and the corresponding sign refer in the first place to an identification of two symbols designating the same object, numbers being then seen as a particular case of this situation.

> Subject: Teaching mathematics in elementary school.
> (...)
> Mathematics teaching in elementary school wants to respond henceforth to the imperatives that result from prolonged compulsory schooling and from the contemporary evolution of mathematical thought.
> (...)
> The ambition of such teaching is thus no longer essentially to prepare pupils for active and professional life by making them acquire techniques for solving problems catalogued and suggested by "everyday life", but to ensure

> them a correct approach and a real understanding of the mathematical notions linked to these techniques.
> (...)
> The activities so far called "calculation" remain, of course, essential, but since they now constitute only a part of the mathematical activity of children, it is appropriate to refer to the subject of the curricula as "Mathematics".[1]
>
> 3. - Comparing two numbers
> 3.1. - Use of the sign "="
> Generally speaking, when we write a=b, it is because the symbols a and b refer to the same object.
> In particular a number can be expressed in different ways:
> Examples: 6; 2 × 3; 4 + 2; 8 - 2; 24 ÷ 4 are designations of the same number.
> This gives the right to write:
> 6 = 6
> 6 = 2 × 3
> 2 × 3 = 4 + 2
> etc. [2]

We are here at the heart of the reform of modern mathematics, which intends to place the foundations of mathematics (in particular set theory) at the beginning of learning in a will of epistemological parallelism (Gosztonyi, 2015, p.133) between the structure of "la mathématique" (in the singular) as it was called in France at that time and the organization of mathematics education. This ambition of the reformers is verified when we closely analyze the programmatic choices. Indeed, if we refer to the emblematic work of the Bourbaki (1977, p.38) group, and especially to its *Theory of Sets* volume, a simple reading of the summary of the book shows that equality is only introduced in the fifth and last chapter entitled "Equality Theories". To put it briefly, an equality theory is a part of mathematics in which one can define a notion of equality that realizes what one intuitively expects from it, i.e., that two objects are said to be equal when they have the same properties. Bourbaki (1977, p.39) thus explains that "by misuse of language, when a relation of the form T = U has been proved in a theory T, we often say that the terms T and U are the same or are identical (translation by the author)". He also demonstrates in this chapter the theorem x = x, which fully ensures the consistency of the notion of equality. In a rigorous and modern exposition of university mathematics, equality is thus presented well after the notions, considered more fundamental, of membership (denoted ∈) and nonmembership (denoted ∉), which appear from the first pages of the book. As is seen later in the analysis of the filmed session, this choice of an organization of school concepts that sticks as closely as possible to academic mathematics has important consequences for students' experiences.

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[1] Circulaire n° IV 70-2 du 2 janvier 1970, translation by the author.
[2] *Idem*, translation by the author.

The video *Égalité d'objets* is the result of experiments that preceded the writing of the modern mathematics reform programs by only a few years. It shows an approach very similar to the one presented in the textbook *Les Mathématiques du cours préparatoire*, written by Guy Brousseau and published by Dunod in 1965. In a recent text, Brousseau (2015) returns to this book, which he himself describes as a workbook aimed at helping 6-year-old students, who generally cannot yet read, discover fundamental notions in mathematics such as numbers and first operations. By providing numerous drawings and adapted examples, Brousseau intended to promote the use of mathematical knowledge in context directly by the children, as opposed to a lecture by the teacher. From this perspective, the notion of equality (and consequently the sign =) must be constructed by the students as the right solution to a problem among several alternatives. As Brousseau reminds us, this way of approaching equality is very different from another one, more common in teaching before 1960-70, which made equality the expression of a result at the end of an intuitively temporal process. For example, to a first number (or magnitude), one adds a second number (or magnitude), and the result is then this number (or magnitude). The sign = becomes here the marker of this action, which goes from the data to the result. Based on the merchant game, the session presented in the film *Égalité d'objets* endeavors to reconcile mathematical rigor with manipulation of objects close to the everyday life of the pupils. This last point prefigures the expectations of the 1970 curriculum in which one can read that "In spite of its desire to be an approach, the most correct and the most precise possible, of some fundamental concepts, abstract by nature, the teaching of mathematics in elementary school remains resolutely concrete (translation by the author)". It is therefore a question of conducting learning periods that do not boil down to an exposition of abstract concepts but that propose the implementation of activities involving games and manipulations that allow the notions to come alive. The challenge is to ensure that the whole is coherent and that the students learn what they are supposed to.

Description and analysis of the filmed session

After a short credit sequence reminding us of the partner institutions, the director, the title of the session and the level, the film places us directly in the classroom as a kind of guest observer at the back of the room. Later, the use of several cameras (probably two) offers other points of view, close-ups on the blackboard, on some students or on the teacher. The activity proposed to this first-grade class is a variation of the merchant game in which students must order clothes to dress a doll. For this session, the students in the class are divided into three groups, each facing a board, so that only the middle group (Group 3) has a view of all three (see Figure 1).

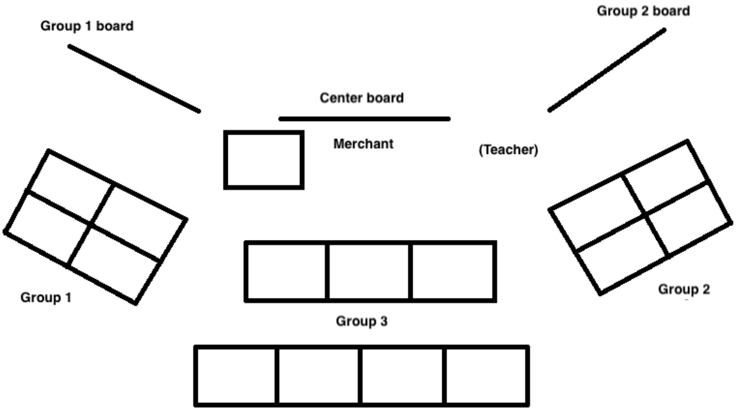
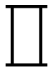


Figure 1: General classroom organization for the activity. Throughout the session, the teacher stands to the right of the center board.

For the two groups on the edges (Group 1 and Group 2), a list of doll clothes associated with various symbols is displayed on the board; the middle board remains available for common phases. For these first graders who are just beginning to read, the objects are represented only by drawings. A set of labels with the symbol is placed on a chair in front of each board. Each of these two groups has its own doll to dress. The scene is completed by assigning a particular role to a student, the shopkeeper, who has boxes containing one of the doll's clothes (shoe boxes, therefore opaque) marked with a symbol.

Doll clothes	Symbols set n°1	Symbols set n°2
Shoes		
Vest		
Dress		
Socks		
Pants		
Coat		

For this game, the shopkeeper is presented as deaf and dumb, so she communicates with the other students only through symbols. Each group, in turn, must order a piece of clothing from the merchant by presenting her a card with the correct symbol. The activity makes sense because the merchant's boxes are only marked with the symbols of set 1. Thus, while Group 1 can easily order a suit, Group 2 does not have the right set of symbols. At this point, the middle group (Group 3) intervenes and, having an overview of the whole situation, must help the second group students to obtain the outfit they want. The objective is to establish a correspondence between the symbols on list 1 and those on list 2. Through the game with the merchant, the children are encouraged to indicate that two different signs designate the same object and must in this way construct the notion of equality between two abstract symbols (the shapes of the symbols having no relationship either to each other or to the garment in the game). In addition to this first knowledge aimed at the situation, the necessity of a sign to write equality is also highlighted. It is obviously the introduction of the sign = that constitutes a socially shared designation of this new relationship constructed by the pupils.

Teacher-student interactions and the mathematics content: a short example

In the first part of the session, after the presentation of the game, the teacher asks the students in the first group to order an outfit. A student offers to do so and, realizing that her oral request is unsuccessful, she gives the label with the symbol for socks (symbol set 1) to the shopkeeper. The shopkeeper serves her customer by giving her the corresponding box. The customer opens the box. She is satisfied; she has obtained the socks. After the first order by the first group, the second phase of the activity is centered on the outfit order of the second group. As mentioned above, this second group has a different set of symbols than the first. However, the merchant's boxes are labeled only with the first symbols. As soon as the first order is placed, a hiatus arises because, although they have chosen the right symbol (set 2), the pupils of the second group cannot obtain the desired outfit; the shopkeeper does not understand their symbol. The problem of relating pairs of abstract symbols structures the session. At this point, the center group (the helping group) intervenes. Children propose to put side by side the cards representing the two symbols (⊡ the one from set 1 and ⍁ the one from set 2) corresponding to the dress (the object ordered). The targeted knowledge on the notion of equality is carried by the situation, its different elements and its different rules. The pupils are then able to construct both the meaning of this notion (two abstract symbols have the same function/meaning) and a first representation (spatial matching of the labels). What remains to be done is to evaluate this solution (does the matching of the labels make it possible to solve the problem?), then to give this spontaneous construction a real status of knowledge, and finally to identify the socially shared representation of this knowledge (i.e., the sign =).

The next phase of the session, which is largely taken over by the teacher, is dedicated to identifying the knowledge at stake, including both the meaning of the notion itself and its representation. By recalling the situation that just played out, the teacher leads the students to focus their reflection on the signs. The challenge lies in finding a verbal and graphic designation to translate the equivalence of the two signs. Here starts a discussion between the teacher and her students, the transcription of which is given below (S_i indicates a pupil and T the teacher, indications on what the actors do are given in square brackets):

> T: [The symbol ⊡, understandable by the merchant, was written on the center board by a student. The students try to figure out how to make the merchant understand the sign ⧄. Throughout the discussion, the teacher holds the card with the symbol ⧄ in front of her]. Can we change the sign on the board? [a student S_1 want to give an answer] Therefore, come and let's see, come and try to show what you mean.
> S_1: [erase the first symbol ⊡ and draw the one from set 2 ⧄]
> T: Press the chalk, we won't be able to see well, wait, I'll play it again... Here.
> S_1: [draw $\notin$ to the right of the symbol ⧄]
> T (surprised and sketching a smile): However, what do you mean with that?
> S_1: [draws the symbol ⊡ to the right of the $\notin$ sign, which gives the board ⧄ $\notin$ ⊡, some inaudible words]
> T: To exchange this one for that one and why?
> T: Therefore, you put that, right. What did you mean by that? Explain to us what you meant.
> S_1: It belongs to this sign.
> T: [pointing to the symbol ⊡] It belongs to this sign... it's not quite right... Elizabeth do you have an idea [another student comes to the board] These two signs do have something yes... It might not be quite right...
> S_2 (another student in the class): It's the same.
> T: Therefore, what do you mean?
> S_2: That every sign, they are the same.
> T: They are the same, so we'll find a drawing maybe to say that they are the same.
> S_2: [begins to draw under the signs, interrupted by T]
> S_3 (another student in the class): They are equal.
> T: They're equal, well I'll tell you how it would be written if they were equal. [T draws in the middle of the board] The sign of this dress [drawing ⊡], of this catalog...
> S_s (several students at the same time): Is equal...
> T: It's the same as this one [drawing ⧄] because it does the same thing, it points to the same dress. Well, I [drawing the sign = between the two] will put it like that. Now you can go and order your dress [the merchant serves the customer well].

> Film[3] *Égalité d'objets*, between 8:59 and 11:03. Transcription and translation by the author.

In this part of the session, the teacher resumes her role to complete the intended learning. At the end of the previous phase, the students seem to have understood that the two symbols designating the dress in each of the games have the same role, but this knowledge has not been put to the test, through action, in the situation, or institutionalized. Guided by the objective of correct learning of mathematical formalism, the teacher seeks above all to have students produce designations (verbal and graphic) for equality. The students then fail to represent this knowledge, i.e., to encode it in writing. In the above transcript, one student, after writing one of the two symbols on the board, draws the sign ∉ to her right and then the second symbol (i.e., ⧄ ∉ ⊡), which makes no mathematical sense because these are objects and not sets. How can we understand this student proposal? By looking at the first-grade textbooks published in 1970, we can notice that, in accordance with Bourbaki's mathematical construction recalled in the beginning, the year of the first grade begins with the learning of set notions, then numbers, etc. The sign of belonging does not appear in textbooks, activity sheets or teacher guides. In the filmed session, the student's erroneous proposal shows that she has already encountered the symbol for not belonging. This mathematical content marked by the structure of modern mathematics may confirm the 1969 date present in the record. Indeed, to produce resources quickly, in connection with the IREMs, many experiments were conducted before the publication of the curricula and the textbooks conforming to them. This film on the notion of equality seems to be a good example. The work on the implementation of the main principles of the reform is reflected in a somewhat incongruous but very significant way.

The fourth and final part of the filmed class session consists of a training phase on the use of this new symbol. Extending the pretextual situation of the game of the merchant, the task consists of writing on the board the different equalities between the abstract symbols of the two sets. With the labels, the students succeeded perfectly in making the correspondence table, which they checked as they went along by taking clothes from the boxes. At this stage, the symbol = remains linked to the context of the activity, and its extension to mathematical objects in general does not appear in this film, which is only a class "moment".

A session, a corpus, a new look

Thanks to the preservation work conducted by the *Cinémathèque Centrale de l'Enseignement Public* (University of Paris 3 - Sorbonne), the session on equality could be resituated within a corpus of nearly 80 films devoted exclusively to mathematics (and of nearly 300 films if we consider all disciplines). It is important to note that

[3] Video available with French and English subtitles: https://youtu.be/xkZfLkICvr0?t=524

these films are indeed intended for training, as shown by the presence at the beginning of some titles of a text explaining the issues involved in watching them:

> This lesson does not claim to be a model to be imitated. It is the expression of an example from an actual classroom, an object to be studied by beginners or experienced teachers.
> It invites critical analysis from those who participate in its use.
>
> Text at the beginning of certain films of the serie "Témoignages pédagogiques" directed by Raoul Rossi. Translation by the author.

According to the creators of the films, the sessions do not constitute models to be applied, but they give examples of learning time conducted in real classes. These films, in spite of numerous cuts to make them compatible with the medium format (reels of 30 minutes maximum), retain a certain sincerity. Thus, in several films, similar to the one that has just been analyzed, students' mistakes are preserved as well as the way the teacher is led to use them for the construction of learning. Several other elements show that there is a promotion of a certain idea of mathematics education, centered on an active pedagogy that takes on its full scope during this period too. These approaches, which could be described as innovative, contrast quite strongly with the traditional image of the heritage of the modern mathematics era. Indeed, the descriptions of the teaching of mathematics in the 1970s are often an occasion of a severe judgment, as one can read under the pen of Houzel (2004, p.57) in the *Gazette de la Société Mathématique de France (SMF)*:

> At the end of the sixties, a movement to reform secondary mathematics education was launched in most countries, and this movement unfortunately claimed to be based on Bourbaki. What came out of it was what was called modern mathematics, the harmfulness of which is no longer in doubt. However, it is unfair to put the burden on Bourbaki, whose only fault was to lose interest in the problem after having let Dieudonné make a rather dangerous propaganda to teachers.

In this text, Houzel explains that one should not confuse the work of the Bourbaki group, which concerns fundamental research in mathematics, with the so-called reform of modern mathematics in France, which concerns the teaching of mathematics at different levels from elementary school to university. Indeed, with the exception of the much-contested Dieudonné, who was involved in the first pedagogical reflections that prefigured the reform (De Bock, 2023), the mathematicians of the Bourbaki group participated only slightly in what was happening at that time in mathematics education, especially in primary school (Félix, 1985, 1986, 2005). However, can we say of this reform that its "harmfulness is no longer in doubt"? We must undoubtedly qualify this by distinguishing between the contents (and its mastery by the pupils) and the pedagogy. With regard to content, through the example of the construction of the notion of equality, it is clear that the weight of structuralism initiated by Bourbaki is very strong. Mathematical notions but also and above all their organization, follow university textbooks almost to the letter. These recent mathematical concepts did not always have the expected echo among students who were very young. Therefore, there have been many failures. However, these difficulties should not mask the profound changes in the methods of teaching mathematics initiated by the reform. After viewing approximately fifty films in this corpus devoted to the implementation of renewed mathematics, strong pedagogical choices clearly appear: classroom setup in clusters, group work, several pupils at the blackboard simultaneously, acceptance and management of errors, creative collective aids, objects and individual aids to be manipulated, etc. In these training films, in addition to the new mathematical content, modern teaching methods are also presented, which is also clearly evident in the editing. A quantitative analysis conducted on a sample of ten films shows that most of the time, nearly 80% of the time is dedicated to students placed in an investigative situation. The phases of presenting the activity, giving instructions and summarizing are often short, from one to two minutes, which leaves time to see the discussions between the teacher and his or her class. Concerning these last interactions, the films also propose a certain vision of the role of the teacher, which places him or her far from the *ex cathedra* approach. The teachers in our corpus, filmed between 1969 and 1972, speak a lot with the students, invite them to present their results on the blackboard, and consider both right and wrong answers. The autonomy of the students is at the heart of learning, and the teacher therefore tries to set up situations that allow him or her to provide accompaniment in learning rather than solely transmission of preestablished knowledge. Another element that is very present in the various films in this corpus, in connection with the pedagogical choices just mentioned, is the use of manipulatives. In all the filmed sessions, the pupils manipulate objects. These objects can be taken from everyday life (such as acorns, chestnuts, candies, cubes or card games) or commercialized for educational purposes (multibase blocks, logic blocks). Moreover, manipulatives are sometimes created specifically by the teacher for the occasion. It can be simple large posters (with a pointed network, for example) but also cardboard cylinders or even inner tubes with lines and stickers stuck on them. In accordance with the syllabi or prefiguring it in the case of videos dating from before 1970, the films strongly emphasize lively, concrete teaching, with materials adapted to children. With successive reforms, the content changed slightly, but the pedagogical orientation remained.

Conclusion

To conclude, because of the medium itself, the viewing of training films contributes to a new look at the modern mathematics reform. While the reading of official documents and even pedagogical documents of that time can give us a very arid image because it is crushed by an omnipresent formalism, the pedagogical testimonies realized by Raoul Rossi within the framework of the Audiovisual Center of Saint Cloud show us on the contrary classes where students and teachers experience lively mathematics. These sources, which are precious because they are quite rare, give substance to a whole part of a reform whose actors wanted changes in content to be accompanied by changes in methods. As early as 1968, in the Chambery Charter (Charte de Chambéry, 1968), the APMEP emphasized active pedagogy and warned against the excesses of formalism.

> We would like to stress that the introduction of new content in the teaching of mathematics will be ineffective, even harmful, if it is not accompanied by an appropriate pedagogy: active, open, as undogmatic as possible, calling upon group work and the imagination of the children.

For the historian, these intentions, which are aimed at both primary and secondary levels, were already known, but the audio-visual sources had yet to be rediscovered and promoted. Obviously, as in any massive production of resources, the documents currently found are rather uneven. Some sessions are very rich, others seem more artificial, but this is also why they constitute a new source whose study needs only to be deepened. The work undertaken in partnership with the *Cinémathèque Centrale de l'Enseignement Public* contributes, we hope, to enriching in a fruitful way the previous historical studies conducted on this rich and eventful period.

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