The study of modern mathematics in the military course of mathematics (1753-1756) of Pedro Padilla

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Abstract

In 18th-century Spain, military institutions played an essential role in the teaching of higher mathematics. Toward the end of 1750, an Academy of Mathematics was established at the Royal Guards Barracks in Madrid and was ruled by Pedro Padilla (1724-1807?) until it was dissolved in 1760. In 1753-1756, Padilla authored and published his Curso Militar de Mathematicas *[Military Course of Mathematics] for the specific use by this Academy. According to the preface of his first volume, Padilla's primary aim was to show that understanding the basic principles of each branch of mathematics could be useful to not only infantry and cavalry regiments, but could serve engineers, artillery and navy personnel. Besides, the publication of Padilla's course was also an early initiative done so during a time when the tradition of dictating was still very strong. Of the twenty mathematical treatises that Padilla originally intended to develop, only the first five would be published in the end (in four volumes). The treatises IV and V are the most innovative, since they contain the earliest educational works on higher geometry and calculus published in Spain, respectively. The aim of this paper is to explore how the study of higher geometry and calculus was approached by Padilla in his course and, in particular, to identify the sources on which it could be based. The analysis of the treatises IV and V in Padilla's mathematical course contributes evidences of the study of modern mathematics in 18th-century Spain.*

Keywords: Pedro Padilla, Academy of Mathematics, Military Course of Mathematics, higher geometry, calculus

Introduction

In 1711, the first Bourbon King of Spain, Philip V (1683-1746), approved the establishment of the Spanish Corps of Military Engineers. This corps was initially conceived by the Engineer General Jorge Próspero de Verboom (1667-1744) to promote the development of scientific activity for the purpose of modernizing Spain supporting the country's defense and protecting its trade activities, after the stagnation the country had suffered by the end of the Habsburg dynasty (Galland Seguela, 2008). The military engineer was to be a specialist in scientific matters, military engineering, drawing, architecture and mathematics. As a direct consequence, the Barcelona Royal Military Academy of mathematics was founded in 1720 to provide scientific knowledge for military and technical training. This academy was ruled by royal ordinances that established the topics to be taught each academic year, outlined how they should be taught and designated the staff overseeing coursework (Portugues, 1765, VI, pp. 858-925). Two additional military academies were founded in Ceuta (1732) and Oran (1739), along the North coast of Africa, which followed the model of regulations established by the leading

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academy in Barcelona.[1] In 1739 a royal ordinance established regulations with the view of developing a general course program for military training. The headmaster in Barcelona should choose the most useful treatises for each subject to then produce notebooks, which were later dictated by the assistants and copied down by the students, not only in Barcelona, but also in the academies of Ceuta and Oran. Following this ordinance, in 1739 Pedro de Lucuce y Ponce (1692–1779), headmaster of the academy for the periods 1738-1756 and 1760-1779, began working on his mathematical course for military training. This course, completed in 1744, consisted of eight treatises on the main fields of Mathematics, including pure mathematics (arithmetic and geometry) and mixed mathematics (cosmography, statics, hydraulics, architecture, artillery, and fortification). While it would never be published, Lucuce's course was used in the context of the Academy of Barcelona for over forty years and was effectively preserved in the students' notebooks of the time.[2]

In 1717 Philip V established the Royal Guards Barracks in Madrid, mirroring the French *garde du corps du roi*. It was an elitist institution, intended mainly for noblemen and officers. Toward the end of 1750, an Academy of Mathematics was created within the Royal Guards Barracks, as part of the reforms launched by Zenón de Somodevilla (1702 – 1781), the 1st Marquis of Ensenada (Hidalgo, 1991; Lafuente & Peset, 1982; Portugues, 1765, V, pp. 180-184, 187, 196-199). Chief Minister at the time, Ensenada undertook several initiatives to overhaul and modernize training programs within both the Army and the Navy. Despite being a rather small academy, the financial budget of the Academy of Mathematics was much higher than that granted to other larger academies, a fact made evident by its extensive library. Up until its closure in 1760, the Academy was under the direction of Pedro Padilla y Arcos (1724-1807?) (Capel et al., 1988; Cuesta Dutari, 1985, pp. 136–137; Garma, 2002), who in 1753 began publishing his *Curso Militar de Mathematicas, sobre partes de esta ciencia, para uso de la Real Academia establecida en el Cuartel de Guardias de Corps* [Military Course of Mathematics, which in part addresses the mathematical sciences, for use by the Royal Academy established as a part of the Military Academy of the Royal Guards]. The aim of this paper is to explore how the study of higher geometry and calculus was approached by Padilla in his *Curso* and to identify the sources on which it could be based. The analysis of this course contributes evidences of the study of modern mathematics in 18-century Spain.

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[1] The teaching of mathematics at the Barcelona Royal Military Academy of Mathematics has been largely studied in Capel et al. (1988), Massa-Esteve et al. (2011), Massa-Esteve (2014), Muñoz Corbalán (2004), Riera (1975).

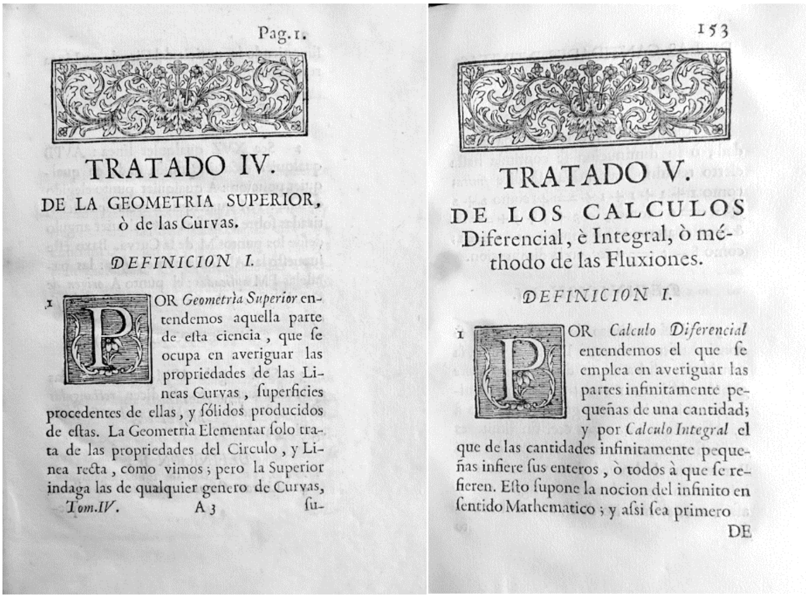
[2] For a thorough account of such notebooks and, in general, of the mathematical aspects of Lucuce's course, see Massa-Esteve et al. (2011).

Pedro Padilla and his *Curso Militar de Mathematicas*

Padilla's primary aim was to show the basic principles of each branch of mathematics, useful to not only infantry and cavalry regiments, but also to engineers, artillery and navy personnel (Padilla, 1753-1756, Preface). Through his work, Padilla introduced what represented an essential change in the pedagogical methods thus far (Blanco & Puig-Pla, 2020). As mentioned above, Lucuce's course was dictated by assistants and copied down by students at the Barcelona Royal Military Academy of Mathematics, hence following the royal ordinances. By making printed versions of his *Curso* available, Padilla aimed to relieve students "from the annoyance of writing, incompatible with their daily duties", so that they could "make greater progress in the studies" (Padilla, 1753-1756, Dedication to the King). The *Curso* was regarded as auxiliary to the teaching program, which placed greater emphasis on lectures and discussions conducted in the classroom (Padilla, 1753-1756, Previas noticias al curso, §17). Padilla presented his work more as a guide, an introduction to all branches of mathematics that could be expanded and completed, if necessary. This approach contrasts radically with the teaching regulations in the military academies in Barcelona, Ceuta and Oran, where teaching assistants should rely exclusively on the notebooks provided and developed by the headmaster, and to not deviate from their content without the headmaster's approval (Portugues, 1765, VI, pp. 906-907). Although teaching through dictation was not completely abandoned at the Academy of Mathematics in Madrid, Padilla's course represented an attempt to standardise the mathematical knowledge to be taught and learnt (Hidalgo, 1991, p. 1954). Maybe his *Curso* was nothing but Padilla's own notes in print, and if so, we could be witnessing a significant transition from "handwritten notebooks" to "printed books" in the Spanish military context (Blanco, 2013).

Among other projects, the Marquis of Ensenada encouraged the production of educational works in Spanish that would raise the level of education in a more uniform manner across all military academies (Capel et al., 1988, p. 161). Padilla's *Curso* can be regarded as an outcome of the reforms carried out at the time by the Marquis of Ensenada. The more so since Padilla's course was approved by Jorge Juan (1713-1773), one of the leading figures of the Enlightenment in Spain. Juan praised the clarity, extension and order with which Padilla treated the different branches of mathematics and consequently he recommended the work as a useful addition to teaching curriculums. This is particularly remarkable given Juan's interest in improving science teaching in Spain, and the substantial level of institutional support he had received to further this mission, notably from the Marquis of Ensenada (Ausejo & Medrano Sánchez, 2010; Garma, 2002).

Of the twenty treatises that Padilla originally intended to develop, only five came out in the end (in four volumes), namely: I. Ordinary arithmetic; II. Elementary (Euclidean) geometry; III. Elementary algebra; IV. Higher geometry, or geometry of curves, and V. Differential and integral calculus, or the method of fluxions. The treatises IV and V in Padilla's course (Fig. 1) can be regarded as the first educational works on higher geometry and calculus, respectively, to be written in Spanish (Ausejo & Medrano Sánchez, 2010; Blanco, 2013 and 2021; Cuesta Dutari, 1985).



Pag. 1.

TRATADO IV.

DE LA GEOMETRIA SUPERIOR, ò de las Curvas.

DEFINICION I.

1 POR *Geometrìa Superior* entendemos aquella parte de eſta ciencia, que ſe ocupa en averiguar las propriedades de las Lineas Curvas, ſuperficies procedentes de ellas, y ſólidos producidos de eſtas. La Geometria Elemental ſolo trata de las propriedades del Circulo, y Linea recta, como vimos; pero la Superior indaga las de qualquier genero de Curvas,

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TRATADO V.

DE LOS CALCULOS Diferencial, è Integral, ò méthodo de las Fluxiones.

DEFINICION I.

1 POR *Calculo Diferencial* entendemos el que ſe emplea en averiguar las partes infinitamente pequeñas de una cantidad; y por *Calculo Integral* el que de las cantidades infinitamente pequeñas infiere ſus enteros, ò todos à que ſe refieren. Eſto ſupone la nocion del infinito en ſentido Mathematico; y aſsi ſea primero

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Fig. 1. First pages of treatises IV and V of Padilla's *Curso*.

Treatise IV: On higher geometry

Padilla's fourth treatise, *On higher geometry, or geometry of curves*, deals with the geometric construction of equations, first order and second order lines (or conics) and the resolution of various geometric problems (including solid problems). That is, the content of this treatise is related to the study and application of the geometry of René Descartes (1596-1650). Mathematical activity in Spain at the end of the 17th century and the beginning of the 18th century was familiar with Cartesian geometry, as shown by the works of Hugo de Omerique (1634-1705), Pedro de Ulloa (1663-1721) or Tomás Vicente Tosca (1651-1723), among others (Dorce, 2017, pp. 8-24; 31-62). According to Navarro Brotons (1997, p. 235), the first text published in Spain that deals, however briefly, with Cartesian geometry is *Elementos de Matemáticas* (1706), by Pedro de Ulloa. In the first volume of his work, Ulloa specifically talks about the algebraic representation of curves and the algebraic theory of Descartes' equations. The second volume was to contain "the principles of algebra and its application to geometry", but it was never published. On the other hand, although it is true that the *Compendio Mathematico* (1707-1715) by Tomás Vicente Tosca (1651-1723) contains the first treatise on conics published in Spanish (volume III), conics are there treated from a geometric point of view, with no equations. Hence, Padilla's treatise IV can be regarded as the first educational work on higher geometry in Spain.

In his fundamental study on the introduction of infinitesimal calculus in Spain, Cuesta Dutari (1985, pp. 124-125) states that Padilla's fourth treatise is mainly based on the first volume of *Elementa Matheseos Universae* by Christian Wolff (1679-1754). Although the influence of Wolff's work cannot be denied, an exhaustive reading suggests other possible sources and influences. The more so since, as I mentioned above, the Academy of Mathematics had a large library furnished with the main reference works on the subject, according to the inventory shown in Cuesta Dutari (1985, pp. 127-133). This section examines and revisits the possible sources upon which Padilla's fourth treatise could rely.

Treatise IV (151 pages, 93 figures) opens with 9 definitions and then is divided in the following sections:

- Geometric construction of equations (§§12-40)
- On the first-order lines (§§41-45)
- On the second-order lines, or first-class curves (§§46-91)
- On the conic sections (§§92-125)
- On other curves (§§126-129)
- On geometric problems (§§130-135)
- On the divisions of a straight line (§§136-149)
- On the regular polygons, and their inscription in the circle (§§150-162)
- On triangles (§§163-174)
- On quadrilaterals (§§175-176)
- On solid problems (§§177-187)

According to Cuesta Dutari (1985, p. 125), the structure of this treatise resembles that of the *Traité analytique des sections coniques* (1707) by Guillaume de L'Hospital (1661-1704). However, unlike Padilla, L'Hospital's treatise opens with the study of conics, leaving for the final books the "problèmes indéterminés", the construction of equations and "problèmes déterminés". A quick comparison with Descartes' *Géométrie* reveals a similar content organization in Padilla's treatise (except for the sections on polygons and geometrical problems). Hence, the first section of Padilla's Treatise IV seems to be connected with Descartes' book I (*Problems Which Can Be Constructed by Means of Circles and Straight Lines Only*); the next four sections deal with curves, as in Descartes' second book (*On the Nature of Curved Lines*); and the last section clearly refers to Descartes' third book (*On the Construction of Solid and Supersolid Problems*). It is worth pointing out that L'Hospital did not use the term "solid problems", but "problèmes déterminés". The structure and the explicit use of the term "solid" seem to indicate that one of Padilla's sources could have been Descartes' *Géométrie*. Not an unlikely hypothesis, given the fact that one can find the *Commentaires sur la Géométrie de M. Descartes* (1730), written by Claude Rabuel (1669-1729), in the library inventory of the Academy of Mathematics (Cuesta Dutari, 1985, pp. 127-133).

When it comes to the contents, on several occasions Padilla's approach differs from Wolff's *Elementa*. To illustrate my point, next I will discuss the first definitions in Padilla's fourth treatise.[3] First, according to Definition I higher geometry deals with curved lines, surfaces and solids, whereas the elementary geometry addresses straight lines and circles (Padilla, 1753-1756, IV, §1). In the course introduction, higher geometry is a branch of geometry, divided into "elementary, spherical and higher". This classification is not exactly the one presented by Wolff in his *Elementa Analyseos*, which is divided into two parts: I) finite and II) infinite. The former is in turn divided into two sections: 1) "De arithmetica speciose" and 2) "De algebra". The second section contains the chapter "De algebra ad geometriam sublimiorem applicata" (part I, section II, chapter VI), that is to say, higher geometry is studied in the section dealing with algebra. However, Padilla's approach to the general division of mathematics, which he discusses at length in the preface, appears to be a direct echo of D'Alembert's *système figuré* in the *Discours Préliminaire* of the *Encyclopédie* (D'Alembert & Diderot, 1751, I), according to which geometry is divided into elementary and transcedental, the latter dealing with any kind of curves. There is still another example to support my point: according to Padilla, algebra was divided into elementary algebra and infinitesimal algebra, the latter in turn being divided into differential calculus and integral calculus (Padilla, 1753-1756, Previas noticias al curso, § 21), as displayed in the *système figuré*. Meanwhile, in Wolff's division, calculus is contained in part II of the *Elementa Analyseos*, the one dealing with infinite elements. Despite not referring to it explicitly, Padilla's classification could illustrate the early reception and circulation of the *Encyclopédie* in Spain (Blanco, 2013, p. 774).

In the second definition Padilla defines the axis of abscissas and the origin as follows:

> DEFINITION II. Let *XVZ* be any line, *AVPB* any given undefined straight line in any position, *A* any chosen point on it, *PM* any parallel lines drawn on it at any angle from the points *M* on the curve. Under this assumption, the *AP* are called the *abscissas*, the parallel lines *PM* are the *ordinates*, the point *A* is the *origin of the abscissas*, and the line *AP* is the *axis of the abscissas*.[4]

[3] For an extended study of this treatise, see Blanco (2021).

[4] "DEFINICIÓN II. Sea *XVZ* qualquier linea: *AVPB* qualquier recta indefinida dada en qualquier posicion: *A* qualquier punto elegido en ella: *PM* qualesquiera rectas paralelas tiradas sobre la misma en qualquier angulo desde los puntos *M* de la curva. Baxo este supuesto las *AP* se llaman *abscisas*: las paralelas *PM aplicadas*: el punto *A origen de las abscisas*: y la recta *AP eje de las abscisas*." (Padilla, 1753-1756, IV, §2)

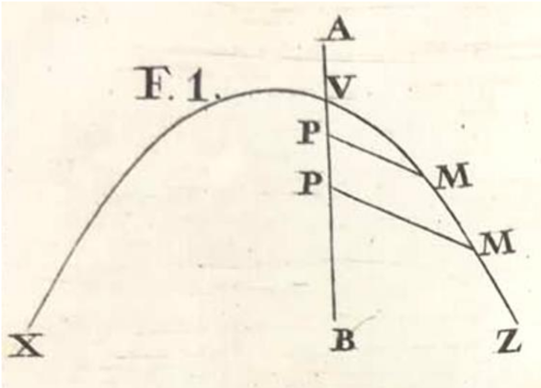

Fig. 2. Diagram used by Padilla to illustrate the definition of axis of abscissas and the origin (Padilla, 1753-1756, IV, Plate I, Figure 1).

Padilla regards the origin *A* as any point on a given straight line *AB*, which represents the axis of abscissas. From Fig. 2 it appears that the origin does not belong to the curve. In my opinion, here Padilla's approach differs considerably from Wolff's (1732, I, §§370-371).[5] First, Wolff defines the diameter of a curve as the line *AD* that bisects the parallel lines *MM* at *P* (Fig. 3), which he calls the axis when it cuts them at right angles. Then, the vertex of the curve is the point *A*, from which the diameter starts. Finally, Wolff defines the abscissa as part of the diameter, or another line, to which the curve refers, included between the vertex or a fixed point, and the point where the semi-ordinate intersects the curve. Here the line *MM* is regarded as the ordinate, whereas the line *PM* is the semi-ordinate, as shown in Fig. 3. By contrast, what Wolff defines as the semi-ordinate would be the ordinate according to Padilla. It is clear, then, that for Wolff these elements are linked to the curve. In fact, he defines diameter and vertex of a curve, before defining abscissas and ordinates.

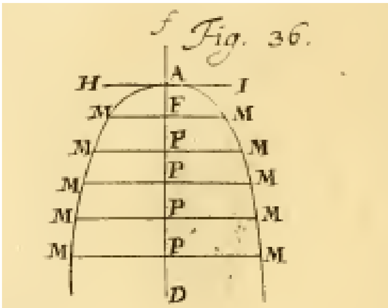

Fig. 3. Diagram used by Wolff to illustrate the definition of diameter and vertex (Wolff, 1732, I, Tab. III, Figure 36).

[5] There are numerous editions of the Latin version of Wolff's work from 1713 to 1754. For this study, I have used the edition of 1732, vol. I, pp. 235-416.

Padilla's definitions seem to indicate a more general approach, in line with Reyneau's approach, who also establishes the origin as a fixed point from which a given line originates within the plane where the curve is located:

> In the plane where each curve is a straight line whose position is given on the plane, and a fixed point from which it starts, which is called its origin, also given. This straight line is called the line of *abscissas*. We assume an infinity of other lines, all parallel to each other, starting from every point on the curve and intersecting the line of *abscissas*. These are called the *ordinates*. The portion of the line of *abscissas* from the origin to the ordinate that terminates it is the ordinate. To abbreviate, each abscissa and its corresponding ordinate are called the *coordinates*.[6]

In the subsequent definitions, Padilla tackles the classification of curves. In definition IV Padilla defines continuous curves as those for which the relationship between abscissas and ordinates is always the same ("siempre una misma"), unlike for the discontinuous curves (Padilla, 1753-1756, IV, §4). Then, in definition V, he defines algebraic curves as those whose relationship can be expressed algebraically, that is, the powers of abscissas and ordinates are rational, unlike the transcendental curves, for which the powers are irrational:

> DEFINITION V. Algebraic curves are those whose relationship between abscissas and ordinates can be expressed algebraically, while transcendental curves lack this circumstance. In algebraic curves, the powers of abscissas and ordinates are rational, whereas in transcendental curves, they are irrational.[7]

These definitions are not to be found in Wolff's *Elementa*. Yet, Padilla's definitions turn out to be quite similar to the ones provided by Gabriel Cramer (1704-1752) in his *Introduction à l'analyse de lignes courbes algébriques* (1750), a work to be found in the inventory of the library of the Academy of Mathematics. For instance, the way Cramer defines regular lines matches the definition of continuous curves given by Padilla:

> Regular lines, on the contrary, are those that are described according to a constant law that determines the position of all their points. There is a uniform property that applies to all points on the same regular line.[8]

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[6] "On suppose sur le plan où est chaque courbe une ligne droite dont la position est donnée sur le plan, & un point fixe d'où elle part, qu'on nomme son origine, qui est aussi donné. Cette droite se nome la ligne des *coupées*: on suppose une infinité d'autres droites toutes paralleles entr'elles qui partent de tous les points de la courbe, & et vont toutes couper la ligne des coupées, on les appelles les *ordonnées*; la partie de la ligne des coupées depuis l'origine jusqu'à l'ordonnée qui la termine, est la coupée de cette ordonnée; &, pour abreger, on nomme chaque coupée & son ordonnée correspondante, les *coordonnées*." (Reyneau, 1708, II, preface, vj)

[7] "DEFINICIÓN V. Curvas Algebricas se dicen aquellas cuya relacion entre abscisas, y aplicadas se puede expressar algebricamente; y transcendentes las que carecen de esta circunstancia, esto es, que en las primeras sean racionales las potencias de las abscisas, y aplicadas, è irracionales en las segundas." (Padilla, 1753-1756, IV, §5)

[8] "Les lignes régulières sont, au contraire, celles qui sont décrites suivant une loi constante qui détermine la position de tous leurs points. Il y a quelque propriété uniforme qui convient également à tous les points d'une même ligne régulière." (Cramer, 1750, §2)

Besides, Cramer also identifies algebraic with rational curves, and transcendental with irrational curves, as Padilla does in definition V, as quoted above:

> These types of curves are called transcendental, mechanical, or irrational, to distinguish them from those that can be represented by algebraic equations and are therefore called algebraic, geometric, or rational curves.[9]

Later in his treatise, Padilla classifies the curves according to the degree (or dimension) of their equation. If the equation is of the second degree, the lines are of the second order ("orden"), or curves of the first class ("género"), and similarly for higher degrees. How did Wolff present this issue? He refers to a curve of the first class ("generis") when the equation has dimension two (Wolff, 1732, §382). Likewise, there were authors, like Reyneau, who used "genre" for both lines and curves, that is to say: "lignes du second genre" or "courbes du premier genre" (Reyneau, 1708, VIII, §354). Padilla's classification, according to order of a line and class ("género") of a curve, seems to agree with the one presented in the *Encyclopédie*:

> It should be noted that a *curve* of the first class is the same as a second-order line because straight lines are not considered curves, and a third-order line is the same as a *curve* of the second class. Therefore, *curves* of the first class are those whose equation involves two dimensions, while curves of the second class involve three dimensions. Curves of the third class involve four dimensions, &c.[10]

This was in fact the terminology used by Newton in his *Enumeratio linearum tertii ordinis* (written around 1676 and published in 1704 as an appendix to his *Opticks*), as mentioned also in Cramer's work:

> Mr. Newton distinguishes between the orders of lines and the types of curves. Since the first order only includes straight lines (…), he refers to second-order lines as curves of the first kind, third-order lines as curves of the second kind, and so on.[11]

The aim of this section was to highlight some of the differences between Padilla's treatise IV and Wolff's *Elementa* concerning the definition of the axis and origin of abscissas, the division of mathematics and curve classification, among others. The next section moves on to the study of calculus in Padilla's treatise V.

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[9] "Ces sortes de courbes sont apellées transcendantes, méchaniques, ou irrationelles; pour les distinguer de celles qu'on peut représenter par des équations algébriques, & qu'à cause de cela on nomme courbes algébriques, géométriques, ou rationelles." (Cramer, 1750, §8)

[10] "Il faut observer qu'une *courbe* du premier genre est la même qu'une ligne du second ordre, parce que les lignes droites ne sont point comptées parmi les courbes, & qu'une ligne du troisieme ordre est la même chose qu'une *courbe* du second genre. Les *courbes* du premier genre sont donc celles dont l'équation monte à deux dimensions; dans celles du second genre, l'équation monte à trois dimensions; à quatre, dans celles du troisieme genre, &c." (D'Alembert & Diderot, 1751, p. 4: 382)

[11] "Mr. Newton distingue les ordres des lignes & les genres de courbes. Comme le premier ordre ne renferme que la ligne droite (…), il appelle courbes du prémier genre, les lignes du second ordre, courbes du second genre, les lignes du troisiéme ordre, & ainsi de suite." (Cramer, 1750, p. 53, footnote). Here it is worth pointing out that at the end of this footnote, Cramer claimed that, since this terminology was very awkward, in his work he would use indifferently curves or lines of n^{th} order.

Treatise V: On calculus

At a time when the teaching of mathematics at Spanish universities was of rather low quality, if not inexistent, the teaching of calculus was mildly undertaken by the military institutions and the Jesuits (Ausejo & Medrano-Sánchez, 2010; Berenguer, 2021; Garma, 2002). There are a number of evidences of the reception and uses of calculus in the first half of the 18th century. Hence, as early as 1717, the Spanish student Francisco de la Torre Argáiz defended a collection of public theses connected with calculus at the University of Toulouse, under the supervision of the French Jesuit Jean Durranc. Later on, Jorge Juan, together with Antonio de Ulloa (1716–1795), took part in the expedition of 1734 to measure the length of a degree of the meridian arc at the Equator, organized by the Académie des Sciences de Paris (Garma, 2002, pp. 334–336). As a result of their voyage, in 1748 Juan and Ulloa published *Observaciones astronómicas y physicas hechas en los reinos del Perú* (Astronomical and physical observations made in the Kingdoms of Peru), which included some applications of the calculus. Regarding the teaching of calculus, in 1752 Padilla presided over some examinations in the Academy of Mathematics of the Royal Guards (Conclusiones, 1752). The fact that a number of these examinations dealt with the calculus and higher geometry clearly indicates that both subjects were being taught in this academy before Padilla's *Curso* came out. However, none of these evidences were neither educational books, nor broad expositions of the elements of the calculus.

Padilla's treatise V, *On differential and integral calculus, or the method of fluxions* (104 pages, 38 figures), covers the following topics: infinite quantities, fluxions, integral calculus or inverse method of fluxions, tangent to curves, maxima and minima, quadrature, rectification of curves, solids and their surfaces. From the examination of the contents of his treatise on calculus, it is clear that Padilla was prone to the method of fluxions (Newtonian approach), yet by means of the d-notation (Leibnizian approach). In fact, he combined both approaches, Leibnizian and Newtonian. On the one hand, the first section dealt with infinite and infinitely small quantities, a text that recalls Fontenelle's *Éléments de la Géométrie de l'infini* (1727). On the other hand, the heading of the second section happened to be "On fluxions". In §20 of this section Padilla defined lines, surfaces and solids in terms of their kinematic generation. From this definition, an instantaneous fluxion, or difference, is nothing but the extension generated in a given time by the motion: "the fluxion, or motion *Mm* generated in a very small instant of time" (Padilla, 1753-1756, V, §20) is the difference, or instantaneous fluxion (Fig. 4).

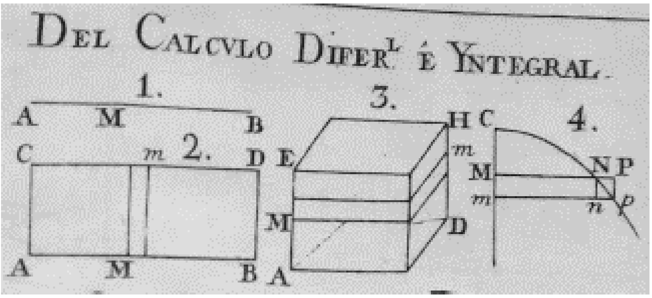


Fig. 4. Diagrams used by Padilla to illustrate the definition of fluxion (Padilla, 1753-1756, V, Plate VI).

As we will see, these definitions were more than likely inspired by Maclaurin's *Treatise of Fluxions* (Maclaurin, 1742, I, §§10–11). Padilla mistook differences with fluxions, the former being used by Leibniz, the latter by Newton. This mix-up, not unusual at the time, can also be found in Maclaurin (1742, II, §723). Yet, in §53 Padilla claimed that the Leibnizian approach was rather abstract, whereas Newton's position, being based upon geometry, appeared to be more natural. This dependence on geometry introduces another element of eclecticism in Padilla's *Curso*, since we have to recall that Padilla had considered calculus as a branch of algebra in his preface. A bit further down, Padilla defines more precisely what the basic element of calculus is the instantaneous fluxion, or difference, that occurs when the element begins to flow, "not after having started to flow, not before, but at the first instant of the beginning" (Padilla, 1753–1756, V, §53). Despite his marked sympathy for Newtonian concepts, Padilla used the Leibnizian *d*-terminology throughout the treatise V, without mentioning the characteristic dot notation of fluxions. This was not unusual in the Spanish context, as Berenguer (2021) shows in his analysis of the treatise of fluxions (1757-1759) by Tomàs Cerdà.

Another illustrative example supporting this view concerns the determination of maxima and minima. According to Padilla, the method of maxima and minima was the most useful among the applications of differential calculus, as he stated at the end of the section on maxima and minima, and here he explicitly referred to Maclaurin's *Treatise of Fluxions*:

> Among the many uses of the differential Calculus there will hardly be another more useful than that of the maximis & minimis: we could show here more instances; but since these are based upon the principles of mechanics, and others which have not been treated so far, their application will take place in due time, and the abovementioned will suffice to understand the rules, which have to be kept in mind, to avoid making the same mistakes that most of the authors have supplied, and that was discovered in the way we have shown here by the celebrated Maclaurin in his Treatise of Fluxions.[12]

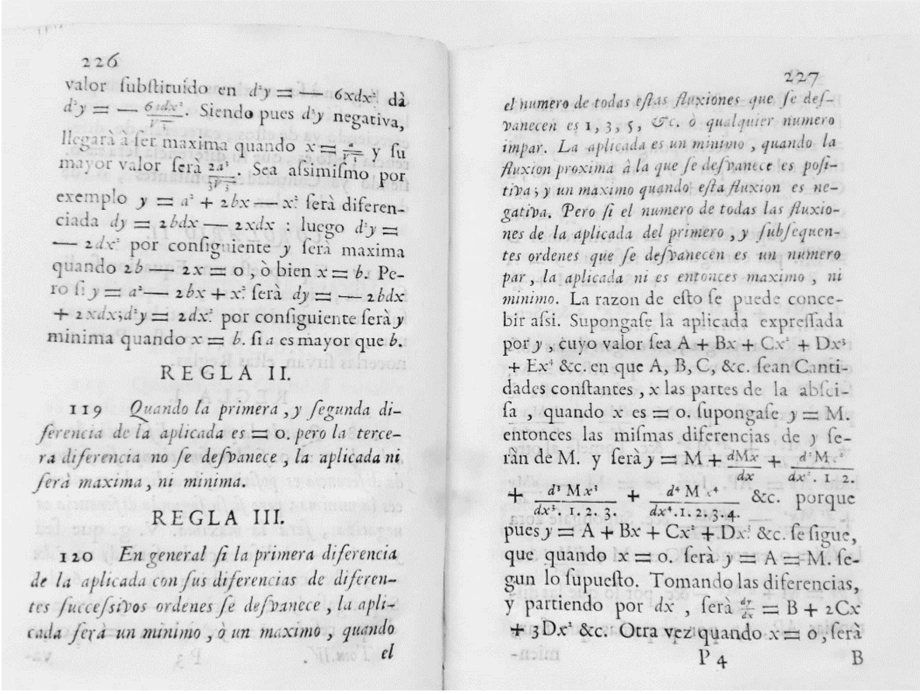
[12] "Entre los muchos usos del Calculo diferencial apenas havrá otro mas util que el de maximis, & minimis: pudieramos traer aquí otros muchos exemplares; pero como estos se fundan sobre

It was Colin Maclaurin (1698–1746) who, in his *Treatise of Fluxions* (1742), presented one of the first algorithms for characterising maxima and minima. From his series expansion, based in turn on Taylor's series (Feigenbaum, 1985), Maclaurin developed his rule by means of the successive fluxions of a quantity. Doubtless Maclaurin's characterization of maxima and minima marked a cornerstone in the path of what we consider today to be the standard derivative tests for determining maxima and minima. His method for maxima and minima had a significant influence on subsequent authors. In the *Institutiones calculi differentialis* (1755) Leonhard Euler (1707–1783) derived the basic criteria for a function to have a maximum or a minimum value in terms of the first and second derivatives, almost identically to Maclaurin's approach, which in turn played a significant role in the works of Joseph Louis Lagrange (1736–1813) and Augustin Louis Cauchy (1789–1857).

In the quotation above, Padilla openly acknowledged the influence of Maclaurin's *Treatise of fluxions*. After having introduced a few particular cases in rules I and II, in rule III (§120) Padilla carried out the general determination of maxima and minima in much the same way as Maclaurin did in book II, §858 (Fig. 5). Padilla went on to prove this rule, following Maclaurin's proof step by step, only he transcripted it into Leibnizian notation. Hence, he expressed the series expansion of a given ordinate, *y*, as shown in Fig. 5 (next page), where $y = M$ for $x = 0$.

It is very telling that Padilla chose this new and sophisticated method in his book, not to be found in other works of reference. In the inventory of the library of the Academy of Mathematics of the Royal Guards in Madrid one can find international masterpieces on calculus (Cuesta Dutari, 1985, pp. 127–133). In particular, the library held a copy of the French version of Maclaurin's treatise of 1749 written by Esprit Pézenas (1692–1776), a French Jesuit with a strong interest in Newtonianism. Most likely Padilla relied upon this version, as the fact that a typographical error in Pézenas's translation (§860, second volume) can be found in Padilla (1753-1756, V, Rule IV, §121) seems to prove, as Cuesta Dutari already pointed out (Cuesta Dutari, 1985, p. 126). The inventory of the library of the Academy of Mathematics of the Royal Guards Barracks also included books on calculus written from a Leibnizian standpoint, such as the *Analyse des infiniment petits* by L'Hospital (1696). So after all, it was Padilla's choice to adopt Maclaurin's work, consciously leaving aside other standpoints.

principios de mecánica, y otros que no se han dado, se hará la aplicación à su tiempo, bastando los referidos para comprehender las reglas, que se deben tener bien presentes, à fin de no caer en los mismo errores que los mas de los autores han dado, y que descubriò en la forma que llevamos dicho el célebre Maclaurin en su Tratado de Fluxiones." (Padilla, 1753–1756, V, §128)



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valor ſubſtituido en $d^2y = - 6xdx^2$ dà $d^2y = -\frac{6adx^2}{\sqrt{3}}$. Siendo pues d^2y negativa, llegarà à ſer maxima quando $x = \frac{a}{\sqrt{3}}$ y ſu mayor valor ſerà $\frac{2a^3}{3\sqrt{3}}$. Sea aſsimiſmo por exemplo $y = a^2 + 2bx - x^2$ ſerà diferenciada $dy = 2bdx - 2xdx$: luego $d^2y = - 2dx^2$ por conſiguiente y ſerà maxima quando $2b - 2x = 0$, ò bien $x = b$. Pero ſi $y = a^2 - 2bx + x^2$ ſerà $dy = - 2bdx + 2xdx$; $d^2y = 2dx^2$ por conſiguiente ſerà y minima quando $x = b$. ſi a es mayor que b.

REGLA II.

119 *Quando la primera, y ſegunda diferencia de la aplicada es = o. pero la tercera diferencia no ſe deſvanece, la aplicada ni ſerà maxima, ni minima.*

REGLA III.

120 *En general ſi la primera diferencia de la aplicada con ſus diferencias de diferentes ſucceſsivos ordenes ſe deſvanece, la aplicada ſerà un minimo, ò un maximo, quando el*

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el numero de todas eſtas fluxiones que ſe deſvanecen es 1, 3, 5, &c. ò qualquier numero impar. La aplicada es un minimo, quando la fluxion proxima à la que ſe deſvanece es poſitiva; y un maximo quando eſta fluxion es negativa. Pero ſi el numero de todas las fluxiones de la aplicada del primero, y ſubſequentes ordenes que ſe deſvanecen es un numero par, la aplicada ni es entonces maximo, ni minimo. La razon de eſto ſe puede concebir aſsi. Supongaſe la aplicada expreſſada por y, cuyo valor ſea $A + Bx + Cx^2 + Dx^3 + Ex^4$ &c. en que A, B, C, &c. ſean Cantidades conſtantes, x las partes de la abſciſa, quando x es = o. ſupongaſe $y = M$. entonces las miſmas diferencias de y ſeràn de M. y ſerà $y = M + \frac{dMx}{dx} + \frac{d^2Mx^2}{dx^2.1.2.} + \frac{d^3Mx^3}{dx^3.1.2.3.} + \frac{d^4Mx^4}{dx^4.1.2.3.4.}$ &c. porque pues $y = A + Bx + Cx^2 + Dx^3$ &c. ſe ſigue, que quando $x = 0$. ſerà $y = A = M$. ſegun lo ſupueſto. Tomando las diferencias, y partiendo por dx, ſerà $\frac{dy}{dx} = B + 2Cx + 3Dx^2$ &c. Otra vez quando $x = 0$, ſerà

P 4 B

Fig. 5. Rule III in Padilla (1753-1756, V, §120).

Concluding remarks

The treatises IV and V in Padilla's mathematical course were the first printed works in Spain for the teaching of higher geometry and calculus, respectively. This contribution pointed to some possible sources that Padilla could have used in the elaboration of these treatises. On the one hand, studying Padilla's treatise IV allows us to identify key differences concerning the contents on higher geometry in Wolff's *Elementa Matheseos*. Instead, Padilla's work appears to share the views of other works of reference, such as Reyneau (1708), Rabuel (1730), Cramer (1750) and the *Encyclopédie*, regarding the division of mathematics, the classification of curves, structure or basic definitions, among others. On the other hand, the elaboration of Padilla's treatise V illustrates a story about choices. From all the available sources at hand, Padilla chose Maclaurin's work on fluxions as a source for the part on differential calculus in his treatise V. Therefore, we can conclude that, in the elaboration of treatises IV and V, Padilla followed an innovative and pedagogical approach, both at a methodological and conceptual level.

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