The teaching of "indirect methods" by Gabriel Lamé, from the École polytechnique of Paris to the Institute of engineers of St Petersburg (1818-1827)

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Abstract

Gabriel Lamé was a mathematician, he is well known for his work in mathematical physics, which he did at the time he became a professor at the École polytechnique in 1832. In this paper, we are interested in the young Lamé, engineer and mathematics teacher, about what he called "indirect methods" in geometry. Our main aim is to analyze the relations between his various teaching assignments and his mathematical conceptions. We examine two different contexts. Firstly, in a textbook written while he was a student at the École polytechnique in the years 1817-1818. Secondly, in the context of his and Émile Clapeyron's teaching at the "Institut du corps des ingénieurs des voies de communication" [Institute of Transportation Engineers] in St Petersburg from 1820 to 1827.

Keywords: geometric methods, problem solving, problems of maxima and minima, rational mechanic, Émile Clapeyron, Gabriel Lamé, École polytechnique, Institut du Corps des ingénieurs des voies de communication de Saint-Pétersbourg.

A pedagogical and epistemological context in Paris

Lamé was born in Tours in 1795 into a petit bourgeois family. He attended the Lycée Louis-le-Grand in Paris before entering the École polytechnique in 1814. He was dismissed from this school in 1816 for "insubordination", and he supported himself by giving private courses of mathematics. He was allowed to enter the final exams of the École in the following year, and he was admitted to the École des Mines in 1818 as a student.

Gabriel Lamé's textbook on problems and methods (1818)

Lamé was ranked third in the entrance examination to the École Polytechnique, so he was able to prepare his students to solve the problems given in preparatory classes and in the examination. On the basis of this experience, he wrote in 1818 a textbook, entitled *Examen des différentes méthodes employées pour résoudre des problèmes de géométrie* [Examination of the different methods used to solve problems in geometry], in which he set out his ideas on geometry and its teaching.

In fact, with his *Examen*, Lamé initiated a new type of textbook that developed in the XIX[th] century (Moussard, 2015). As he explained in the introduction, the aim

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is twofold: (i) "to classify problems according to the greater or lesser similarity of their means of solution"; (ii) "to arrive at a finite compound of different means that could be considered general in view of their numerous applications"(Lamé, 1818, p. 6)[1]. He justified the order that goes from problems to methods by writing: "It is partly with a view to perfecting the problems of Geometry, that mathematicians have enriched Analysis with the calculations which today make up the exact sciences" (Lamé, 18818, p. 1). Adopting this order was also perfect for preparing his students for the entrance examinations to the Grandes Écoles, for which it was important to apply mathematics and to solve problems (Belhoste, 2002).

In the introduction to the textbook, Lamé distinguished between analysis and synthesis, explaining that the former is the guide for inventors, while the latter allows the destruction of doubts. The distinction between synthesis and analysis is present in Greek geometry. In Pappus of Alexandria's *Collection*, for instance, they are defined as two types of consequence, synthesis proceeding from axioms to theorems, while analysis proceeds from theorems to axioms. In 1641, Descartes made a different distinction: for him synthesis is indeed the Euclidean order from axioms to theorems, but analysis is the path provided by the method of problem solving, that is, by his algebraic method (Barbin, 2006, pp. 280-284).

Lamé, following Descartes explained that we must have recourse to the two paths: "Analysis thus illuminates synthesis; the synthesis, in turn, illuminates the passages, which analysis has, so to speak, only touched" (Lamé, 1818, pp. 8-9). He exemplified analysis by the Cartesian method of solving geometric problems by reducing them to the solution of algebraic equations. He also emphasised "the constant agreement of Algebra with Geometry, an agreement that allows calculus to be entrusted with the task of discovering new theorems" (Lamé, 1818, p. 6).

He studied the practice of algebra and symbolic calculus very closely, insisting on the visualisation of equations, the choice of notation and the symmetry of the writing of equations (Barbin, 2009). But he wrote in favour of geometry:

> "Which should be admired more, calculation, which begins its work with confidence and almost always ends with the answer to the question, or geometry, which starts without promising anything and sometimes returns with the solution of the problem, the solution of several others that were not asked of it and that it has collected along its way?" (Lamé, 1818, p. 19)

This sentence is quoted by Michel Chasles in his book *Rapport sur les progrés de la géométrie* [Report on the progress of Geometry], where the geometer also stressed on new mathematical results obtained by the young Lamé in his textbook (Chasles, 1867, pp. 46-47).

Lamé's sources: teaching and order of invention

Lamé justified the order that proceeds from problems to theories by asking: "In the teaching of the abstract sciences, would it not be the best method to follow the path of invention?" (Lamé, 1818, p. 2). The "order of invention," from problems

[1] All translations from French to English are made by the author of the paper.

to theorems, was illustrated and defended in Alexis Clairaut's *Éléments de Géometrie* of 1741, and in Étienne Bonnot de Condillac's *Essai sur l'origine des connaissances humaines* of 1746 (Barbin, 2021a, pp. 4-5). Lamé found this order advantageous for showing the goals of mathematics, but also the coordination of its various branches.

Lamé did not give any details about his sources. But we can mention Sylvestre-François Lacroix, a disciple of Gaspard Monge, who became professor of mathematics at the École Centrale des Quatre-Nations in 1794 and wrote several textbooks. In *Essais sur l'enseignement en général et sur celui des mathématiques en particulier* [Essays on teaching in general and that of mathematics in particular] of 1805, Lacroix devoted 20 pages to "the two methods of dealing with the mathematical sciences, synthesis and analysis". He quoted Condillac and affirmed, as Lamé would do, that "analysis is in general the method of invention", and he advised to "bring together synthesis and analysis, whenever one can use both at the same time" (Lacroix, 1805, p. 240; p. 251).

The young Lamé may have read the *Essais* because Lacroix was an examiner at the École polytechnique from 1809 to 1815, in particular during the year where he was a student at the École.

"Indirect methods" in Lamé's *Examen* (1818)

For Lamé, one proof of the richness and of the generality of Geometry is the help it receives from other sciences which owe at least their growth and clarity to it. Recourse to these sciences leads him to what he called "indirect methods", for example mechanics:

> "Mechanics is undoubtedly the science which promises the most discoveries to Geometry; Statics often borrows its synthesis, its principal theorems, and gives it in return either a new proof of a principle already known, or the solution of a problem whose statement it has translated into its own language" (Lamé, 1818, pp. 83-84).

He used the example of finding the centre of gravity of a triangle, which proves that the lines joining the vertices and the midpoints of the opposite sides intersect at the same point. He wrote that: "Sometimes the science of equilibrium makes a game of the most difficult problems of geometry, and sometimes goes hand in hand with the highest calculations of algebraic analysis" (Lamé, 1818, p. 84).

Lamé explained that the problems of maxima and minima of distances (or sums of distances) are often "the pitfall" of Geometry and Algebra and that many problems would remain unsolved, if the infinitesimal calculus had not taken care of them. But Statics offers an "indirect method" to solve them. This method consists of two steps:

1) Translation because it is one of the advantages of Statics to translate relations between lengths into relations between directions;
2) Transformation because there is no equilibrium that is not due as much to the directions of the forces as to the relations between their intensities.

Lamé concluded: “As this transformation from lines to angles is often very difficult to find by simple Geometry, it is not surprising that it is inferior to Statics” (Lamé, 1818, p. 84).

He took a simple problem of minimum as example: to find a point such that the sum of the distances from this point to three given points is a minimum. He asked to consider three rings fixed at points *A*, *B*, *C*, and a cord attached to a fourth ring *O* which will pass successively: through the fixed rings *B* and *A*, through the movable ring *O* again, and finally, through the fourth fixed ring at *C* (Fig. 1).

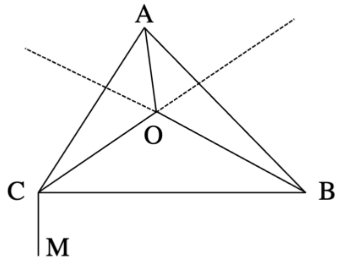

Fig. 1. A simple problem of minimum (Lamé, 1818, pl. 2)

The first step is to translate the minimum problem into an equilibrium problem:

> “It is evident that any force which would pull the cord following *CM*, will be in equilibrium with the resistance of the fixed rings if the sum of the partial cords is a minimum” (Lamé, 1818, p. 85).

Therefore, the point we are looking for is the position of the movable ring *O* in the case of equilibrium. We have to transform then the equilibrium situation into a relation between angles. The cords *AO*, *BO*, *CO* must be equally taut; the ring *O* is therefore pulled in these three directions by equal forces. If there is an equilibrium between these forces, the direction of any one of them must divide the angle of the other two into two equal parts, i.e. the angles *AOB*, *AOC*, *BOC* are equal to each other and to 120°.

Lamé and Clapeyron in the Institute of the Corps of transportation engineers in St Petersburg

We now will examine the context in which Lamé was able to teach “indirect methods” with Émile Clapeyron. Émile Clapeyron was younger than Lamé, born in Paris in 1799. He studied with the Oratorians before being admitted as a student to the École Polytechnique in 1816, and then to the École des Mines in 1818. Clapeyron and Lamé probably met in 1819, as engineering students at the École des Mines.

As students, they wrote two individual papers in the *Annales des Mines*, which indicate rather distant interests, Lamé on the geometric form of crystals in 1819 and Clapeyron on steamships in 1820 (Barbin, 2021b, pp. 16-19).

After graduating from this École, at the end of 1820, they were sent to the “Institut du corps des ingénieurs des voies de communication” [Institute of transportation engineers] in St Petersburg. They were employed to teach mathematics and mechanics respectively. They were also involved in the engineering works of their students, particularly bridges. They remained in Russia until the end of 1831 (Gouzevitch & Gouzevitch, 2009).

The Institute was set up by Augustin de Bétancourt in 1808, using the École polytechnique and the École des ponts et chaussées as "models". Pierre-Dominique Bazaine was responsible for recruiting French engineers, before becoming director in 1824. In 1826, the Institute was presented in the first issue of the *Journal des voies de communication*, the Journal of the St Petersburg Institute, as a place where many "useful truths" could be developed:

> The occupations in which engineers are engaged are so varied and so closely connected, by their object with the branches of the administration of the Empire, that a wise and reasoned criticism of their work must necessarily develop a host of useful truths." […]" Discussion and the mutual exchange of knowledge will make these truths appear in their full light [...], since any improvement brought about by them to the lines of communication will have the effect of making the commerce and industry of the Empire flourish. (*Journal des voies de communication*, 1826, p. 26)

Courses at the Institute

On their return to Paris in 1833, Clapeyron and Lamé described their experiences as teachers in the *Plan d'écoles, générales et spéciales* [Plan of general and special schools], written in relation with Saint-Simonian engineers. Later, Lamé also wrote on their common teaching in the text of a conference given at the Faculty of Sciences of Paris in 1851 (Lamé, 1851).

The courses at the St Petersburg Institute lasted six years and included courses from the preparatory classes, the École polytechnique and the École des ponts et chaussées. Clapeyron and Lamé supervised 600 students during 6 years (Lamé & Clapeyron, 1833, pp. 16-17). They had to teach mathematics, rational mechanics and the theory of machines. Lamé explained in 1851 that they decided to prepare their courses together in order to create a "homogeneous whole" combining theoretical parts and practical applications:

> We had to teach, successively and simultaneously, all mathematics, from the elements to infinitesimal calculus, rational mechanics, the theory of machines, and the construction courses in all their parts. [...] The course on machines and that on construction, which we also taught, helped us by the examples they provided. And these three courses, united in the same hands, formed a homogeneous whole, where mathematical rigour dominated, and from which empiricism was scrupulously banished (Lamé, 1851, p. 227).

In 1833, Clapeyron and Lamé wrote on their students about the teaching of "transcendental mathematics" that included infinitesimal analysis:

> When they [the students] reached transcendental mathematics, the progress of almost all of them suddenly stopped in such a way that all the efforts of their teachers, who could only force them to learn by heart series of calculations and formulae whose meaning and scope they did not

> understand, were in vain [...]. But in the applied sciences, in the construction course for example, the nationals [Russian students] resumed their rapid and progressive progress. Many of them reached an extraordinary degree of perfection, especially in the art of linear drawing" (Lamé & Clapeyron, 1833, pp. 17-18).

Linear drawing consisted of geometric constructions. As, they also wrote, Clapeyron and Lamé believed that it was necessary to know the "natural dispositions" of each individual, in order to assign the kind of education that suited them. Thus, they could consider that geometric constructions and graphic methods would suit young people who excelled in linear drawing and had difficulties with analysis. And so, the talents of their students played a role in their choice of teaching. And this choice of teaching played a role in their own scientific research.

"Indirect methods" in Clapeyron and Lamé's works

Clapeyron and Lamé had to teach theoretical parts of mathematics (geometry, analysis and mechanics) but they also had to follow the practical work of their students. From 1823 to 1831, they worked and wrote on subjects related to their teaching duties, such as the stability of vaults or the construction of suspension bridges. A characteristic of their writings is the simplicity of the means used and of the results obtained. They insisted in their writings on the choice of the methods used by them and on the generality of the solution (Barbin, 2021b, p. 42). This last characteristic, present from the formation of the tandem, can be attributed to Lamé, as the author of the textbook on methods.

The 1823 "Mémoire sur la stabilité des voûtes" [Dissertation on the stability of vaults] is a good example of the use of mechanics as an "indirect method" to solve a problem. This paper, signed by "Lamé et Clapeyron", was the result of an order for St. Isaac's Cathedral in St. Petersburg, which was built from 1818. A vault had been planned for the cathedral, but doubts had been expressed about the possibility of such a construction, and they had been asked to "indicate the precautions to be taken to ensure its stability" (Lamé and Clapeyron, 1823, p. 789).

Since Philippe de la Hire's *Traité de mécanique* of 1695, some fifty memoirs and several works had dealt with vault constructions, but the methods used varied (Becchi & Foce, 2002). For example, in *La science des ingénieurs* of 1729, Bernard Forest de Bélidor relied on geometry to determine the equilibrium conditions of a barrel vault, while Charles Bossut used differential calculus in the "Recherches sur l'équilibre des voûtes" of the *Cours de mathématiques* of 1802.

In 1823, Lamé and Clapeyron reduced the equilibrium condition of the vault to a mechanical problem, that of the equilibrium of two levers. This allowed them to geometrically characterise a breaking point B of the vault (Barbin, 2021b, p. 23). They then translated this condition into a very simple geometric construction: "The breaking point *B* is such that the tangent to the curve *AC* at this point *B* intersects the horizontal line passing through the vertex *K* of the arch at the same point as the vertical line passing through the centre of gravity of the mass *BCKH* tending to detach" (Fig. 2) (Lamé & Clapeyron, 1823, p. 797).

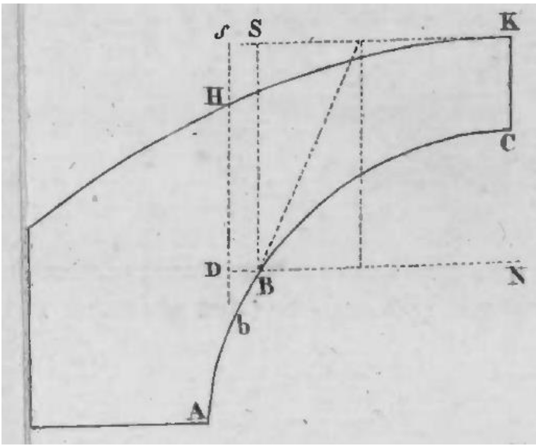

Fig. 2. Construction of the breaking point (Lamé & Clapeyron, 1823, pl. V)

The simplicity of the solution and of the construction may be related to the fact that the vault constructions gave rise to an "application course", given by the tandem at the Institute, on "the pressure of vaults and on the tracing of their rupture joints", for students who were more skilled in geometry than in differential calculus (Lamé, 1851, p. 228).

The "indirect method" for a problem of least distances (1827)

In 1827, the tandem "Lamé and Clapeyron" published the paper "Mémoire sur l'application de la statique à la solution des problèmes relatifs à la théorie des moindres distances" [Dissertation on the application of statics to the solution of problems relating to the theory of least distances] in the *Journal des voies de communications*. They wrote at the beginning:

> There is a singular and remarkable analogy between problems of statics and those of the theory of least distances, an analogy which makes it possible to solve the latter of these problems, by means of procedures of a new kind. We propose, in this memoir, to prove the analogy of which we have just spoken, and to explain the mechanical solutions which follow from it for a multitude of interesting questions (Lamé and Clapeyron, 1827, p. 26)

The mechanical solution of the "theory of least distances"

The tandem specified that the general problem of the "theory of least distances" is to determine the positions of points such that the sum of the lengths of the segments connecting these points multiplied by given coefficients is as small as possible. Then they explained that the general problem is solved "if the algebraic sum of the infinitesimal variations of the lengths multiplied by the coefficients is zero". This general equality, which we denote by (1), is known as the "principle of least distances".

They wrote that this equation follows from the "principle of virtual velocities" if we assume that the points are attracted to each other by forces respectively proportional to the given coefficients. The "principle of virtual velocities" appeared in Jacques Bernoulli's 1725 paper, and it was made explicit by Jean d'Alembert in 1743.

Siméon Denis Poisson expounded this principle in his *Traité de Mécanique* (1811), which followed his lectures at the École polytechnique. Clapeyron and Lamé were certainly aware of this *Traité* in 1827. In his textbook, Poisson considered infinitesimal displacements and stated that if forces P, P', P'', etc. are in equilibrium then the sum of these forces multiplied by the virtual velocities p, p', p'', etc. (considered as oriented segments) is zero (Poisson, 1811, pp. 231-233). So, we have the equation named (2):

$Pp + P'p' + \text{etc.} = 0.$

Consequently, there is an "analogy" between the two principles, "least distances" and "virtual velocities" because they can be identified as corresponding to similar equations (1) and (2) after their translation into the infinitesimal calculus.

The tandem remarked that mechanical systems which materialise the situation would be "of long and difficult use in practice". But they added: "One will be compelled to agree that in the state of imperfection in which algebraic analysis is still found today, the mode of solution here is the only one suitable for the question proposed"(Lamé and Clapeyron, 1827, p. 30)

For instance, for solving the case where all the points are given, except for one, which it would be necessary to determine, it is sufficient to place a vertical rotating pulley at each fixed point and to wind around the pulley a wire from which is suspended a weight proportional to the coefficient corresponding to that point, and to attach the free ends of all the wires to one and the same movable ring (Fig. 3). Then the system, left to itself, will come to rest in such a way that this ring will occupy the position of the point sought. The determination of this point is concretely obtained by the mechanical system and not by a computation.

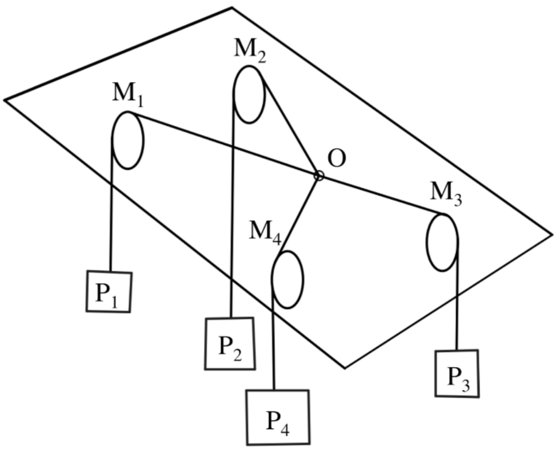

Fig. 3. The mechanical solution

This "indirect method" can be compared with the "funicular balance" of the *Éléments de mécanique statique*, a textbook of Charles Camus of 1752, which makes it possible to obtain an unknown weight Q from a known weight P. These weights are attached to points B and C of a rope suspended at two fixed points A and D. When equilibrium has been achieved, two cords are attached at A and D to mark the intersections N and M of the extensions of AB and CD with CQ and BP (Fig. 4). By measuring CN and BM, we obtain Q by using the proportion $CN / MB = P / Q$.

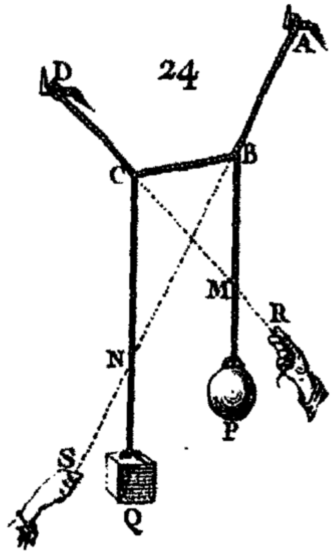

Fig. 4. The "funicular balance" of Camus (Camus, 1752, p. 40)

Applications of the "indirect method"

In their paper, Lamé and Clapeyron insisted on the numerous applications of the "theory of least distances":

> The choice of political or commercial centres of communication, the supply of armies, and even military art in general, the establishment of mines, factories, and the supply of roads, give rise to problems of least distances, which it would almost always be impossible to solve with the aid of algebraic analysis alone, and which will be completely solved by the procedures we are going to develop. (Lamé and Clapeyron, 1827, pp. 26-27).

The mechanical solution can therefore be applied to many problems, such as the problem of locating a factory to process metal ores extracted from different mines located in given places, to achieve the "the greatest possible economy in transport costs".

They give various applications, such as finding the locations of supply centres and army headquarters, or finding the location of "the capital of a new empire" according to the locations of other cities and neighbouring states, etc. They justified the mechanical solution by writing:

> We ask the reader not to lose sight of the fact, before passing judgement on the subject of this memoir, that it gives a means, if not exact, at least very approximate, of solving an infinite number of problems which it would be impossible to undertake by algebraic analysis, both because of the complication of the equations to which they would lead, and because of the difficulty of expressing in the calculation of totally discontinuous data. (Lamé and Clapeyron, 1827, pp. 26-27).

This "indirect method" was also very suitable for students who were intimidated by infinitesimal analysis. It enabled them to obtain, though approximately, "useful truths". The interest in this type of application may have been linked to the tandem's attendance of the Antoine Raucourt's "Cercle de philosophie et d'économie politique" [Philosophy and Economics Circle] in St Petersburg. Raucourt was polytechnician and engineer of bridges and roads, who became a colonel of the Institute of Engineers of St Petersburg. He arrived in Russia in 1821, where he met the Saint-Simonian Prosper Enfantin (Gouzevich & Gouzevitch, 2009). He returned to Paris in 1827 and he published in 1829 a paper "Sur les moyens à employer pour l'entretien et la conservation des routes" [On the means to be employed for the maintenance and conservation of roads] (Raucourt, 1829).

The paper of Clapeyron and Lamé is considered as the "earliest contribution" to local equilibrium in economic and logical contexts (Franksen and Grattan-Guiness, 1989). In particular, with the problem of applied mathematics known today as "the depot problem" in operational research and taught to students at universities.

The use of the principle of virtual velocities had other benefits for the teaching of the tandem. In 1851, Lamé commented on the three common courses of Clapeyron and Lamé on mathematics, mechanics and machine theory:

> In this school [Institute], the time devoted to scientific studies was more limited than it is in France; for example, it was necessary to cover the whole of rational mechanics in thirty lessons; this was very little. In order to make the best use of the time allotted, we have more or less reduced the course to the knowledge of the principle of virtual velocities, and that of living forces, multiplying their applications to numerous subjects. (Lamé, 1851, p. 278)

Restricting the principles to two made it possible to construct rational mechanics as a theory and to increase the number of practical applications. We can note that, in order to present the principle of virtual velocities as a fundamental basis of mechanics, Bazaine would give a proof of this principle, in a booklet published in St-Petersburg in 1832 (Bazaine, 1832).

Charles Sturm's comments: on the originality of the tandem

In 1827, Charles Sturm was a young researcher who worked in a laboratory at the École polytechnique of Paris and was winner of the "Prix de l'Académie des sciences". In a two-page review of the *Bulletin des sciences mathématiques* of 1829, he indicated "the other interesting and numerous applications they [Lamé and Clapeyron] make of their method of mechanical solution" (Sturm, 1829, p. 328). He also wrote that he had already noticed "the use of statics to solve problems of this kind" in an 1823 paper (Sturm, 1823) of the *Annales de mathématiques pures et appliquées,* a journal for mathematicians and teachers (Gérini, 2002).

In fact, in his "Recherches analytiques, sur une classe de problèmes de géométrie dependant de la théorie des maxima et minima" [Analytical research on a class of problems in geometry depending on the theory of maxima and minima], he linked these problems to the equilibrium situation of a point M which is acted upon by forces (Sturm, 1823, p. 113). But the problems were solved with analytical calculations and were far from the "method" of the tandem. The tandem was certainly aware of this paper, and we can imagine that they had it in mind when they wrote about the "complication of the equations" (Fig. 5).

$$
\begin{aligned}
&\{\Sigma[P\mathrm{Cos.}(p,x)]+\Sigma[Q\mathrm{Cos.}(q,x)]+\Sigma[R\mathrm{Cos.}(r,x)]\}\mathrm{d}x\\
&+\{\Sigma[P\mathrm{Cos.}(p,y)]+\Sigma[Q\mathrm{Cos.}(q,y)]+\Sigma[R\mathrm{Cos.}(r,y)]\}\mathrm{d}y\\
&+\{\Sigma[P\mathrm{Cos.}(p,z)]+\Sigma[Q\mathrm{Cos.}(q,z)]+\Sigma[R\mathrm{Cos.}(r,z)]\}\mathrm{d}z\\
&-\Sigma\{Q\mathrm{d}s\mathrm{Cos.}(t,q)\}\\
&+\Sigma\left\{R\left[\frac{\mathrm{Cos.}(r,z)\mathrm{Cos.}(n,x)}{\mathrm{Cos.}(n,z)}-\mathrm{Cos.}(r,x)\right]\mathrm{d}g\right\}\\
&+\Sigma\left\{R\left[\frac{\mathrm{Cos}(r,z)\mathrm{Cos}(n,y)}{\mathrm{Cos}(n,z)}-\mathrm{Cos.}(r,y)\right]\mathrm{d}h\right\}=0. \qquad (\mathrm{I})
\end{aligned}
$$

Fig. 5. Equation of condition for maximum and minimum (Sturm, 1823, p. 112)

Conclusion: the "indirect methods" from one context to the other

To consider their "indirect methods" as legitimate methods is a characteristic feature of Lamé and Clapeyron's epistemological conceptions of mathematics. The two contexts in which they defend this approach are pedagogical: the training of candidates for the polytechnic examination and the teaching of calculus to students with little experience. However, it is interesting to compare their proposals on "indirect methods" in each of the two contexts in which they were presented and put into practice.

On problems and their applications

The geometrical problem of least distances was not new when it appeared in Lamé's textbook of 1818, but, as we have seen, it became a larger problem in 1827 with new kinds of general applications proposed by the tandem. These applications had to be placed in the context of the "Circle of philosophy and political economy" of St Petersburg, which the pair attended and which was hosted by the Saint-Simonian engineer Antoine Raucourt.

In this context, it was important for Lamé and Clapeyron to provide future engineers with a mechanical approach to the problem that does not require analytical knowledge which the students did not have. This issue was widely debated at this time, also regarding general engineers. In 1829, the Central School of Arts and Manufactures was founded in Paris, on a private initiative, by Alphonse Lavallée, a businessman and shareholder of the Saint Simonian-inspired newspaper. The school aimed at the training of general engineers for the emerging industry at a time when the "Grandes Écoles" rather trained state engineers.

On "useful truths"

The solution of 1818 was based on a "translation" thanks to an argument in statics, while the "analogy" of the solution of 1827 was "proven" by an analytical identification of two principles. The mechanical solution of the tandem is original and, moreover, was considered legitimate by them. This solution can be read in the context of the "useful truths" promoted by the St Petersburg Institute. Moreover, it allows to link together the three common courses of Lamé and Clapeyron, and it meets the students' tastes by combining geometry and mechanics.

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