A STEM PROJECT IN MOUNTAINS

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Abstract. *We present a STEM project based on Mathematics and GeoGebra software to be carried out in a mountain environment. But this project also includes other key aspects such as an intense use of technology and engineering, with measurements using GPS system and theodolites or optical angular measurement instruments, typical of engineering. At the same time, we must introduce the basic scientific concepts of cartography and geodesy, to which we could add those of satellite geolocalization, in order to know how and why these instruments work. The project actually consists of several independent problems with a common conceptual framework, for whose objectives we can offer precise answers and even study the margin of error in them based on the precision of our instruments and the configuration of our real scenario. The problems described here can be easily adapted to various urban or rural settings to extend their use beyond the mountains alone.*

Key words: Trigonometry, GeoGebra., cartography, app, GPS, theodolite, error measurement.

INTRODUCTION

The mountain environment offers an unbeatable setting to enjoy nature, but it might also become an exceptional learning classroom. The project we present covers all aspects that can be part of STEM education, showing how in some types of problems, all these aspects must be set in action simultaneously for a correct modeling and solution of it.

It is a project designed to be carried out as a team, where the teacher plays a very important role in directing, tutoring and facilitating, as a scenographic producer, the ways to use the varied mix of knowledge, technologies and equipment necessary for accomplishing the goal. Designed to take place in the mountains (or in some cases in the outdoors), taking data, models and views in a real setting. A partial work can be done indoors, in the elaboration of the data and its final visualization through GeoGebra applets. They can be a complement to standard curriculum subjects or, alternatively, become part of a specific STEAM curriculum.

MEASURING THE MOUNTAINS WITH TRIGONOMETRY

Our first problem is to measure the distance from a point where we are, A, to an inaccessible (by distance or physical impossibility) point V.

One of the best real application to our problem is in the mountains, measuring, for example, the distance from the point where we are to a far away peak. But it would also be worth the procedure to measure the distance to a landmark on the other side of a river, without crossing it, or at the top of a high tree in a park. The farther the inaccessible point, the more precise the angular measuring device that we must have to use.

In the activity we will have a goal drawn from a real problem that combines:

1. A model for solving with elementary mathematics (Sine Theorem, Cosine Theorem, Thales' Theorem), easily transferable to secondary school students (See Wikibooks, 2006).

2. Data acquisition techniques with precision instruments used in engineering such as theodolites or optical goniometers. You can find a detailed description of these instruments

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in (Nadolinets et al, 2017). Also from smartphone devices such as GPS. These techniques require, by themselves, some mathematical, scientific and technical explanation and allow us to get insight into current technologies such as GPS or topographic measurement. This will be accompanied by a field measurement, i.e. a *field trip for a STEM project.*

3. A data processing and visualization through GeoGebra on a cartographic map image. GeoGebra is well known to secondary school teachers and students in most countries. For a more detailed inspection of the commands and procedures, you can refer to (International GeoGebra Foundation) for a manual on how to proceed. This step will take us along the path of added scientific knowledge of cartography and georeferencing, to present our final solution.

4. Finally, we can add a brief study of how measurement errors due to the instruments used affect the final solution (for a general perspective, see Olusegun Ogundare, 2015).

The result is a project that puts together all the STEM letters. We are going to exemplify, in more detail, a case that takes the measurement from a distant point to one of the peaks of Sierra Nevada (Spain), *Veleta*. We take the measurement from the nearest point to that peak, accessible by road. To do this we must follow some steps, as described in the previous items.

1. The mathematical model and its overlapping to reality

We must first design our model and then identify the actual context on it. For this we will take a new point *B*, accessible by foot from *A*, and in such a way the distance between them could be measured using GPS technology. The farther from A, in the manner of Image 1 (trying to make an isosceles triangle), the better, since higher will be the angle γ, as we will discuss later, the fewer errors the measurement will have. In our case, it must be at least several hundred meters away. What we are doing is transforming the problem of measuring an inaccessible side (at one of its ends) of a triangle, by measuring one side, *AB*, and two accessible angles, α, β, of it. Using the Sine Theorem, we will have that the sought distance:

$$(1) \quad \boldsymbol{AV = (AB \cdot \sin(\beta))/\sin(\gamma)\,;\ \gamma = 180° - \beta - \alpha}$$

2. Data acquisition techniques

To calculate the distance *AB*, between accessible but distant points we will use a dedicated GPS device or the smartphone sensor with a vectorial cartography app. We will use the app *Oruxmaps* for a small charge (https://www.oruxmaps.com/cs/en/), but there are many for this purpose, for example, *GPX Viewer* and *View Ranger* are two of them free of charge, although they have lower navigation performance. *Google maps* is not suitable, since it is not possible to load topographic vector maps, requires online operation (little available in mountains) or preprocessing, and uses geographic coordinates, which require subsequent conversion to UTM coordinates, more didactic, clear and easy for calculation.

Then we will upload a geo-referenced map of our area (we can get it for free from most of the *National Geographic Services*), and we will create waypoints (marked points in the cartography app) at selected points *A* and *B*, reading their coordinates.

In order to make the calculation easy and understandable, from a mathematical point of view, we must explain the GPS configuration options, such as the *reference ellipsoid* (International or Hayford), the *datum,* that must correspond to that of our map (ED50), and the coordinate system (UTM) that reveal the distance in m. to the Earth's equator and the Greenwich meridian.

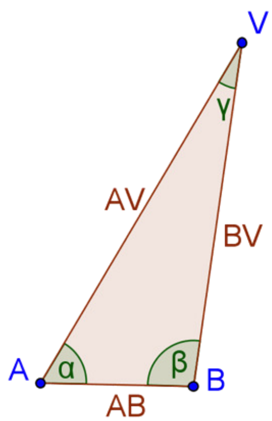


Figure 1: Mathematical model.

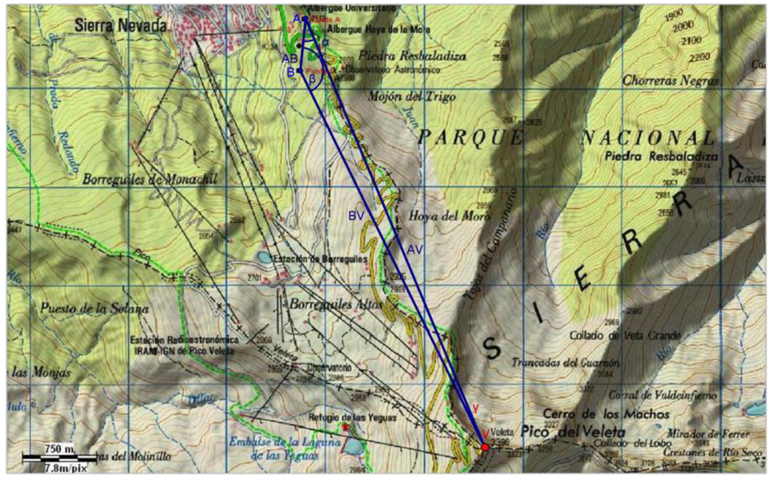


Figure 2: Overlapping the model to our map.

In our case, the coordinates of *A*: 30 S 465821 4105717 and *B*: 30 S 465702 4105148.

To calculate the angles α and β, we will use instruments typical of terrestrial engineering such as the **theodolite** (with precision to the hundredth of a degree) or, if this is not available, from an **optical goniometer** (with usual precision to the tenth of a degree).

In the absence of these resources we can use a **sextant**, an instrument from classical marine navigation, (with less precision) or even a good **alidade compass or digital compass** (for shorter distances and higher γ angle, usual precision up to the degree). For each of them we will need a basic explanation of their opto-mechanical operation and their various leveling, focus and measurement adjustments. We strongly recommend the theodolite by their complete adjustment, superb precision and didactical use.

In our problem, with a theodolite, we can take as values; $\alpha = 32.61°$ and $\beta = 143.19°$.

3. Data processing and visualization with GeoGebra

Although the *A* and *B* coordinate data are on a mountainous surface, is usual to take the distance as the straight line between these two points, and not the distance along the undulating surface of the terrain. In our model we are assuming this convention. Then we have found *AB* then has a value of 581.3 m (using Pythagorean Theorem for a triangle of vertices *A* and *B* at the hypotenuse).

As the value of $\boldsymbol{\gamma}$ is $\boldsymbol{\gamma = 4.20°}$ then, substituing in (1) we have: *AV* = 4755.6 m.

We can also reach this value through the GeoGebra sheet directly. To do this, we must adjust the scale in GeoGebra with the ZoomIn() and ZoomOut() commands and the <Scale Factor> option the grid, so that it coincides, for example in km, with the scale on the image of our loaded map. Then GeoGebra will not only calculate geometrically the final solution (although with less precision than directly, due to higher resolution on the vectorial map than on the raster one) but it will also show a map with all the mathematical objects to scale, much clearer when read.

Figure 3: Theodolite, optical goniometer and Screenshot from our GPS app.

Figure 4: Nautical sextant, alidade compass and GPS device

4. Study of measurement errors

In the measurement process we introduce some error, due to the measurement technique, with which we have to deal. You can inspect. As can be seen, the calculation is very sensitive to the size of γ, since when $\gamma \to 0$, then $\sin(\gamma) \to 0$. So, as this function is in the denominator of expression (1), small variations of this can become in very different values of AV. Hence the importance of taking AB as increasing values when the precision of our measuring instrument decreases. As a complement we can add to the project a small discussion of errors, depending on the precision of the instrument. Assuming an approximation to the tenth of a degree in the measuring instrument, then we can accumulate up to 2 tenths of a degree error in determining the angle γ, and thus:

$\Delta\gamma$ (error)	value of γ	Ratio of $(AB \cdot \sin(\beta))$ in AV
0.2°	3°	13 %
0.2°	2°	29 %
0.2°	1°	117 %

Table 1: Ratio of errors in final value by angle.

Since we have used a theodolite approaching the hundredth of a degree, then each hundredth (+ or -) decreases or increases (respectively) by 11 m. approx. the distance *AV* in our case where $\gamma = 4.20°$. Each tenth of a degree (+ or -) decreases or increases (respectively) by approx. 110 m. the distance *AV*.

We therefore recommend taking a triangle setting that leaves above $3°$ the angle γ, selecting the adequate point *B*, in order to keep the error caused by the angle measurement as low as possible. As a general rule, we should try to arrange a triangle in which points *A*, *B* and *V* are not close to alignment, since this will lead to a larger angle γ, and therefore to a smaller error. But not only the angle γ will determine the error, in addition to the reading error. Also the distance from point *A* to *V* will have an effect: the farthest the point *V* is, the greater the error, following (Ali & Algarni, 2003).

In the measurement of waypoint coordinates with the GPS, and supposing the best acquisition of satellites and location, we could have a minimum error of about 4 *m.* in determining each coordinate. This will give us a total of up to $\sqrt{(x+8)^2+(y+8)^2} - AB \approx 9.5\, m.$ of error in *AB* measurement, being x and y the difference between the E and N coordinates of *A* and *B*. But it should not be forgotten that the error can be triggered in the event of a lack of precision in the acquisition of satellites due to *selective availability*, a rush in the acquisition of satellites, or a deficit in the quality of the GPS chip or signal reception. Although this error is the least influential, the previous table can be completed taking into account all the errors for our angle γ and measuring instruments.

OTHER RELATED PROBLEMS

We can also consider some problems for which we will need the same measuring instruments and similar approaches. You can get more information about these types of problems in (D'Antonio, 2011) and (Maass, 2001).

- One would be to measure the height of a point *V*, whose vertical *h* is inaccessible from an accessible point, *A*. It is based on the previous problem, we only need to measure the angle φ, which raises the *AV* side with respect to the horizon line (ρ is a right angle). See Image 5, where the vision is the usual topographic: point V (peak) is the highest and all points around it are below V.

$$h = AV \cdot \sin(\varphi)$$

- A second is to measure the distance between two points, *V* and *P*, both inaccessible, from an accessible point *A*. For this task we first proceed as in the exposed problem, taking a point *B*, accessible from *A*, and calculating the *AV* and *AP* distances. Image 6.

$$AV = (AB \cdot \sin(\beta))/\sin(\gamma)\,;\; AP = (AB \cdot \sin(\delta))/\sin(\epsilon)$$

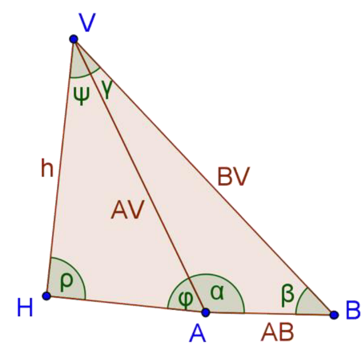
Later, we will continue using the *Cosine Theorem*, which allows us to solve any triangle, known two of its sides, *AV* and *AP*, and the angle between them, σ:

$$VP^2 = AV^2 + AP^2 - 2 \cdot AV \cdot AP \cdot \cos(\sigma)$$

- The third is to calculate the distance between two points, one of them being inaccessible, without using angular measurement instruments. This problem supports many variations, and we can do it by applying the *Thales Theorem,* that states: if in a triangle ABC, a segment is drawn parallel to either side, for example $B'C'$, then triangles ABC and $AB'C'$ turn out to be similar triangles. In Image 7 we can calculate the side BC, inaccessible by measurement with tape (a street with traffic, a river, ...) from AC, AC' and $B'C'$ (can be measured with tape or on foot) and the equality:

$$\frac{AB}{AB'} = \frac{AC}{AC'} = \frac{BC}{B'C'}$$

We add here the schema of the mathematical model and the representation of the solution with GeoGebra for the three problems, without going into the details of calculations:



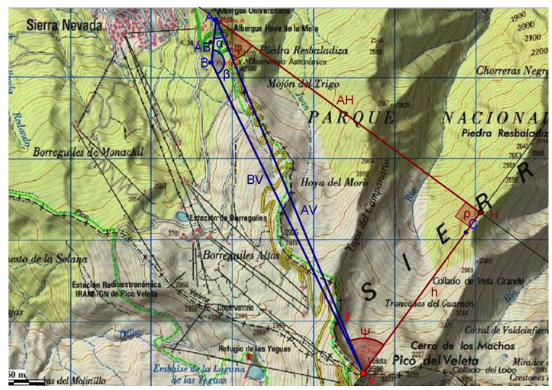
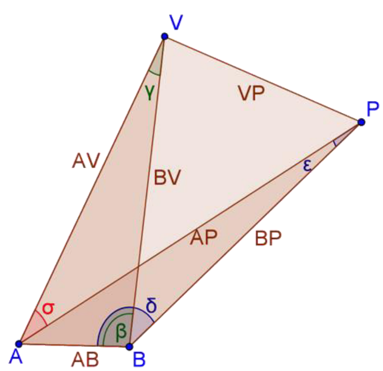


Figure 5: Model and representation for the Height Problem.



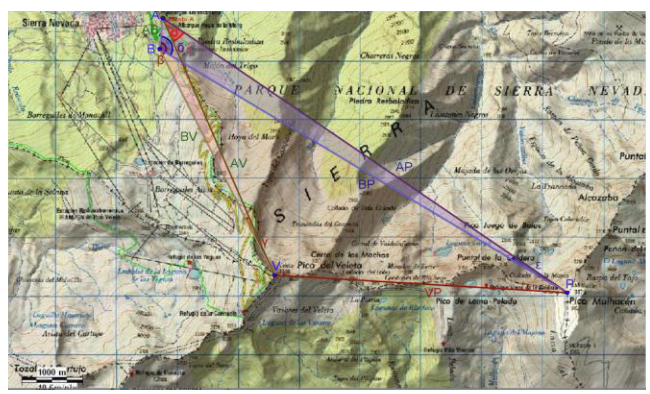
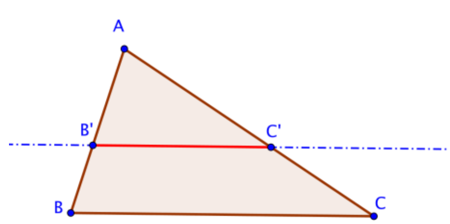


Figure 6: Model and representation for the 2 Peaks Problem.



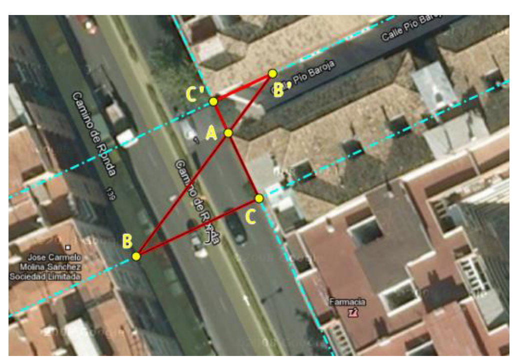


Figure 7: Model and representation for the 3rd problem.

THE EDUCATIONAL PROJECT

This project has been implemented, for three consecutive years, for students in the last years of secondary education. It is part of a broader program to awaken scientific and technological vocations, organized by the Ministry of Science and Innovation in Spain with professors from some universities attached to this project and high school students from all over the country. The project detailed here has been specifically designed by the author in the area of Mathematics and Technology within the scientific program. It has been externally supervised by the FECYT (Spanish Foundation for Science and Technology, a public institution, dependent on the Spanish government). FECYT "works to strengthen the link between science and society through actions that promote open and inclusive science, culture and science education".

The project has also been evaluated, both externally by FECYT, and by the students themselves, achieving a high score in both processes. The students, in evaluation surveys on the content and the methodology taught, have shown their satisfaction with the subjects of the project and its interrelation, as well as with the methodology and, above all, their involvement in it. The project was also finally presented by the students with their own elaboration on the data and materials obtained, before a committee of teachers. Therefore, the entire process has demonstrated high students' participation and great development of STEM learning objectives with very positive evaluation in all phases.

The project has been carried out in 3 scenarios, a classroom for the introduction to section 1, nearby mountain locations previously selected (and an outdoor location in some cases) for section 2 and part of 3 and 4, and finally a computer lab for the final elaboration with GeoGebra in section 3. To annotate, elaborate and study the data acquired in section 2, 3 and 4, students are provided with a form sheet, prepared for this task, in order not to forget any measurements and to organize the computation of the data with scientific calculators or smartphone apps.

Figure 8: Students measuring with goniometer

Figure 9: Students measuring with theodolite.

CONCLUSION

We have exposed a set of related STEM projects in which mathematics, engineering, science and technology interact. They are based on already known trigonometry application problems, but with new instrumentation, data management and scientific representation.

As a result, we have obtained a module in which, apart from the results that are common to expect from many of STEM projects (teamwork, learning to use mathematics in real life, development of critical thinking, ...), students also:

- they must learn to combine different strategies to carry it out in a practical and satisfactory way, combining direct and indirect measures depending on the context.
- they will use tools in which they will learn to assess their precision and performance, and even the crucial influence of errors on the final result.
- they will identify the arrangement and problems of the environment, with a mathematical model and will be able to visualize the problem using GeoGebra and its geometric capabilities, making a representation of it in scientific terms.

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